

**INVESTIGATING MEDIATION STRATEGIES USED BY EARLY
YEARS MATHEMATICS TEACHERS IN MALAWI**

PhD (Mathematics Education) Thesis

By

FRASER PITROS RABSON GOBEDE

MSc. (Informatics)-University of Malawi

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Education, in fulfilment of the requirements for the degree of Doctor of Philosophy
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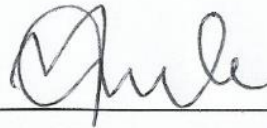
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DECLARATION

I, the undersigned, hereby declare that this thesis is my own original work which has not been submitted to any other institution for similar purposes. Where other people's work has been used acknowledgements have been made.

FRASER PITROS RABSON GOBEDE

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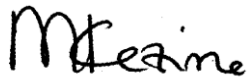
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Date

CERTIFICATE OF APPROVAL

The undersigned certify that this thesis represents the student's own work and effort and has been submitted with our approval.

Signature:  Date: 30 March 2021

Name: Professor Mercy Kazima Kishindo

Main supervisor

Signature:  Date: 30 March 2021

Name: Professor Arne Jakobsen

Supervisory committee

Signature:  Date: 30 March 2021

Name: Professor Reidar Mosvold

Supervisory committee

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DEDICATION

To my beloved family.

You all mean a lot to me!

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ABSTRACT

Based on the sociocultural perspective of the teacher as the main mediating agent in the classroom, the study investigated how teachers work with the four means of mediation; that is, tasks and examples, artefacts, inscriptions, talk and gesture; suggested by the Mediating Primary Mathematics (MPM) framework by Venkat and Askew (2018). The study adopted a qualitative case-study design where the research participants were four teachers from each of the first four classes at a purposively selected school. Data from video-recorded lessons was analysed using the MPM's approach for lesson analysis whereas the interviews and documents were analysed thematically. The study found that the teachers followed the teachers' guide for structuring lesson tasks and selected the mediating examples randomly from the learners' textbooks. The teachers used different artefacts, in particular framed counters for unit counting. The teachers used chalkboard inscriptions for presenting tasks and demonstrating the method to obtain the solution. The teachers' mediating talk for solving addition problems was mainly based on the 'combine and count-all' approach. The teachers' choice of mediating means was generally influenced by the belief that children learn well by doing. These findings have implications for teaching, curriculum materials as well as teacher education and development. The study contributes to knowledge by providing research findings on teachers' mediation of mathematics in early years classrooms in Malawi; and contributes to the MPM framework by exemplifying usage of the framework by a single researcher outside a professional development setup within which it was developed and initially used.

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LIST OF ABBREVIATIONS AND ACRONYMS

MDI:	Mathematical Discourse in Instruction
MPM:	Mediating Primary Mathematics
MS Excel:	Microsoft Excel
MS Word:	Microsoft Word
PD:	Professional Development
PSLCE:	Primary School Leaving Certificate Examinations
SACMEQ:	Southern Africa Consortium for Monitoring Educational Quality
TIMSS:	Trends in International Mathematics and Science Study, formerly called the Third International Mathematics and Science Study.

The following notations were used in transcript excerpts:

T:	Teacher (an utterance or action by the teacher).
C:	Class (an utterance or action by the class).
C+T:	Class and teacher (an utterance or action by both the class and the teacher).
CRECCOM:	Creative Centre for Community Mobilisation (a non-governmental organisation based in Zomba, Malawi). One popular legacy they left during their activities with schools is a special hand-clapping pattern that ends with a cheer).
L1, L2, ...	Learner 1, Learner 2, ... (an utterance or action by a specific learner in the classroom).
R:	Researcher.
<i>[Italicised text]</i>	Actions in an utterance by the teacher or learners.

CHAPTER 1

INTRODUCTION

This chapter discusses the motivation for the study. It outlines some of the major issues associated with the teaching of mathematics in the early years of primary school in Malawi and highlights the need for understanding classroom teaching within the given context. In Malawi, “early years” refers to the first four classes of primary school (Standards 1 to 4). Standard 1 is the equivalent of Grade 1 in most countries.

1.1 Background to the study

Teaching and learning in early childhood is generally considered a complex process that cannot be easily explained by a single theory or perspective (Dunphy, Dooley, & Shiel, 2014). As indicated by Dunphy et al. (2014) the mathematics teacher of young children needs to design classroom tasks that are relevant, motivating, engaging and meaningful to them. Boaler (2016) states that such meaningful learning experiences can enforce a positive mathematical mindset for learners at an early age. However, keeping young learners engaged throughout the lesson presents a challenge for the teacher considering that they have short attention spans, as their minds are easily distracted by extraneous events noticed in their proximity. Ball (1993) shared three dilemmas of teaching primary school mathematics based on her experience as a 3rd-grade elementary mathematics teacher.

The first dilemma experienced by Ball (1993)—regarding the teaching of mathematics to learners in the early years of primary school—is the representation of mathematics content. This requires the teacher to bridge learners’ everyday quantitative understandings with formal

mathematics representations. Choosing a representation or model puts the teacher in a dilemma since a single model rarely captures all aspects of a mathematical idea. The second dilemma mentioned by Ball (1993) involves respecting children as mathematical thinkers. The teacher is expected to induct learners into the mathematics discourse by giving them enough room to express their arguments, hence develop mathematical thinking. This presents a challenge to the teacher because children may fail to say what they are thinking, or they may use words that the teacher may not comprehend. The third and last dilemma discussed by Ball (1993) is about creating and using the classroom as a learning community of mathematical discourse. This requires letting the whole class work together in articulating and refining each other's ideas towards a mathematical goal, thereby building confidence in themselves. Instead of relying on the teacher for verification of results, learners may start to consult their peers within the classroom or other classes (Lampert, 1990). The teacher's dilemma comes in balancing the learning gains and the time spent orchestrating learners arguments—considering the likelihood of leaving learners who were initially correct experiencing uncertainty and confusion after the class discussion (Ball, 1993).

The complexities of teaching mathematics also influence how it is handled as a school subject. Despite the argument that mathematics is the most international (Walshaw & Anthony, 2008) and the largest (Niss, 2012) of all school subjects, there seems to be no internationally agreed effective teaching practices at all levels of the school mathematics curriculum. There are remarkable differences in the way mathematics is taught, making some to conclude that school mathematics appears to be a variant of mathematics. Frobisher (1999) calls school mathematics a distorted version of mathematics, while Boaler (2016) asserts that there is a wide gulf between school mathematics and real mathematics. School mathematics has also been said to be distinct from everyday mathematics or street mathematics (Bishop, 2017; Mosvold, 2008). After considering different understandings of the term 'mathematics', Skemp (1976) identified two

types of mathematics: instrumental and relational, the former appearing easy at first but proving harder with time. Instrumental mathematics is taught as rules without reasons, whereas relational mathematics focuses on both how and why the procedures are carried out that way. Building on Skemp (1976), Gray and Tall (1993) concluded that learners who fail mathematics are doing the harder instrumental or procedural type of mathematics compared to the ones who succeed. Learners who repeatedly fail mathematics eventually disengage their minds in the subject despite the efforts of the teacher (Boaler, 2016). Studies conducted in poorly resourced classrooms in South Africa have shown that mathematics is made meaningful and engaging to learners when the teaching focuses on the structure of the subject as a scientific discipline—characterised by interconnected concepts rather than isolated procedural tasks (Askew, 2019; Ekdahl et al., 2018; Venkat & Askew, 2018).

Teachers of mathematics in the early years of primary school also struggle with tensions and dilemmas between their personal professional convictions of what is good teaching practice against their accountability to policies set-up by the authorities (Roth & Lee, 2007). As put by Rouleau and Liljedahl (2017), this often comes as tension between what the teachers “want to do and what they are asked to do” (p. 155). For instance, in Malawi, the language policy requires the use of vernacular languages in the early years of primary schooling (standards 1 to 4)—the most predominant language being Chichewa. One issue raised by Kazima and Adler (2006) is the counter-intuitive nature of some mathematical concepts, where their use in the classroom differs from everyday life. When such concepts are to be taught to learners with diverse language and cultural backgrounds, they face difficulties to grasp the mathematical meaning. In the Malawi case, Kazima (2008) noted that mathematical terms are not necessarily translated into an equivalent Chichewa register where they carry their conceptual meanings as is the case with the Swahili language in Tanzania. So, teachers work with mathematical words that are just spelt in Chichewa, but the resulting term is meaningless in vernacular.

Teachers are also challenged with the implementation of educational reforms some of which might have a direct influence on their classroom practices. For instance, the current Malawi primary school curriculum emphasizes on outcome-based education, in which learners are expected to display prescribed competencies at the end of instruction—through learner-centred teaching approaches—yet, the teacher is expected to accomplish this goal with minimal technical and material support from the policy-makers and the school (Mtika & Gates, 2010). Teachers' failure to strike a balance between prescribed curricula requirements against what is practically possible has contributed to a general failure in the adoption of learner-centred education in sub-Saharan Africa (Tabulawa, 2013), and Malawi in particular (Mtika & Gates, 2010). In the case of South Africa, Mhlolo (2013) observed that the reforms in the school curriculum were driven by untested assumptions rather than empirically proven research. During a curriculum reform, policy documents may indicate a shift in the methods of teaching, but its implementation in the practice of teaching is often challenging due to resource and cultural constraints. Tabulawa (2013) asserts that if the school system adopts approaches to teaching that are not compatible with the sociocultural context of the school, it will likely not be favoured by both the teachers and the learners. Just like in other countries in the Sub-Saharan Region, Mhlolo (2013) noted that the South African curriculum reform mainly focused on the shift from traditional teacher-led approaches to learner-centred methods—rather than what is made available to learn in the classroom. Regarding the implementation of the above-mentioned curriculum, Hoadley (2012) reported studies in South Africa that found that teachers were using some forms of learner-centred practice yet little learning was taking place. As such, Mhlolo (2013) proposes that researchers in educational practice are in a better position to suggest practical teaching strategies that are grounded on the existing traditional approaches to teaching with the aim of improving what is made available to learn in the classroom.

In addition to the inherent tensions and complexities related to the teaching of mathematics as a subject during the early years of primary school, teachers in Malawi and the Sub-Saharan region also handle classes with high pupil/teacher ratios. The average pupil/teacher ratio in Malawi is 88:1 and is most acute in the early years of primary education where it averages above 100:1 (Ravishankar, El-Kogali, Sankar, Tanaka & Rakoto-Tiana, 2016). This problem places significant pedagogical demands on teachers. For instance, it has been observed that learners in such environments often have notebooks with fewer written work (Graven, 2016; Venkat & Askew, 2018), making it difficult for the implementation of policy on standard expectations on teacher practices, such as marking all learners' work, Graven (2016).

1.2 Statement of the Problem

One of the major perplexing problems in the history of mathematics education has been to understand why learners fail mathematics despite the provision of seemingly adequate teaching and learning resources. Despite the best efforts made by the teacher, there are still some students who possess misconceptions whose origin is difficult to explain. As noted by Sfard (2008), this puzzle has been of major interest by various researchers and theorists throughout the decades, resulting in a proliferation of theories and frameworks for interpreting various approaches for teaching and learning mathematics. Oftentimes, the challenges associated with the teaching of mathematics are seen through low scores attained by learners in the subject. In Malawi, learner under achievement in mathematics has been reported from as early as the first four classes of primary school, based on results from local studies focusing on learner performance. For instance, the 2010 Malawi Early Grade Mathematics Assessment (EGMA) found that about 56% of the 500 Standard 2 learners who participated in the study failed to perform the addition of two single-digit numbers with a sum of less than 10 — which was below the expectation of the Malawi mathematics curriculum (Brombacher, 2011). The

curriculum expects learners to be able to add two numbers with a sum of less than 10 by the time they are completing Standard 1 (Malawi Institute of Education, 2012b). Malawian learners also perform comparably lower than other countries in international standardised assessments in mathematics and numeracy. Surprisingly, the first two tests of teacher knowledge from the Southern Africa Consortium for Monitoring Educational Quality (SACMEQ)—SACMEQ I and SACMEQ II—showed that Malawian teachers possessed sufficient content knowledge to teach Standards 1 to 6 (Ravishankar et al., 2016). As such, the low performance of learners in SAQMEQ tests may not wholly be attributed to teachers' lack of content knowledge, but could possibly indicate their lack of other aspects of their work of teaching mathematics—that is, unpacking their compressed mathematical ideas to the form understandable to the young learners (Ball et al., 2008).

In Malawi, only some 40% of children have the opportunity of experiencing some form of pre-school exposure to basic numeracy through kindergarten, nursery schools, or early childhood development centres (Robertson, Cassity, & Kunkwezu, 2017). This means that about 60% of learners do not have the opportunity of attending some form of pre-school education. This implies that the majority of learners are formally introduced to the concept of number for the first time in Standard 1, at age 6 or more. Since the learners might have some preconceptions of number from their homes and through play, their informal experiences might not be in harmony with what they find in the classroom when they start learning school mathematics. This wide gap between formal school mathematics and their everyday experiences of informal mathematics often makes learners disengage their minds in the subject despite the efforts of the teacher (Boaler, 2016). This disparity presents a challenge on the teacher who has to carefully introduce new mathematical concepts and procedures, some of which possess multiple meanings. Gray and Tall (1994) asserts that learners success in mathematics is highly dependent on their ability to handle the dual nature of mathematical processes and concepts,

giving an example of the term ‘sum’ which could carry different meanings. The sum of 4 and 7 is both the process of adding the two given numbers (symbolised as $4 + 7$) as well as the concept of the result of the addition process. Likewise, the process of counting 4 and the concept of the number 4 share dual meaning. Gray and Tall (2014), therefore, indicate that mathematical tasks can be made more difficult for learners according to the way the teacher handles the process-concept relationship. Taking the example of the process of counting and the concept of number, it is the teacher’s role to ‘reify’ (Sfard, 2008, p. 170) the counting process into its corresponding object: number. If the teacher emphasizes on the counting process, what results is procedural mathematics—termed ‘instrumental’ by Skemp (1976)—whose complexity increases as more and more procedures are added with time. Gray and Tall (1994) illustrate this complexity using the example of finding the sum of two numbers. If the teacher uses the count-all strategy, the sum will be found after carrying out the counting procedure three times, whereas the count-on strategy reduces the counting procedure by applying the concept of number.

Almost all the challenges discussed above have to be handled by the teacher. For the majority of learners in Malawi, the teacher is the sole source of mathematical instruction because many learners do not have access to books and parents may not have adequate education to coach their children at home. The majority of parents in rural Malawi may also not afford private tutoring to supplement their children’s classroom experiences. As such, this study is mainly interested in studying classroom teaching, which is a key determinant to the learning of mathematics in the Malawian context. The main purpose of the study is to investigate the tools and strategies used by teachers during mathematics lessons in the early years of primary school. The idea that mathematical knowledge is socially constructed (Ball, 1993) necessitated the adoption of the sociocultural perspective—which posits that learning is achieved through mediated transactions—with the teacher as the main mediating agent between mathematical

concepts and the learners in the classroom (Venkat & Askew, 2018). In the early years' primary mathematics classroom, the teacher achieves the mediation through sociocultural tools, which Venkat and Askew (2018) categorised into four: tasks and associated example spaces, artefacts, inscriptions, teacher talk and gesture. It was, therefore, deemed worthwhile to understand how Malawian mathematics teachers work with various mediational means during classroom teaching. A deeper understanding of the mediatory practices by the teachers would give insights on the possible strategies for addressing the problem of learner underachievement in mathematics.

1.3 Research questions

The study will seek to answer the following main research question:

How do Malawian teachers of mathematics in early years work with various means of mediation during classroom teaching?

The main question will be answered by asking the following subsidiary questions:

1. How do teachers in the early years of primary school select examples and tasks during mathematics lessons?
2. How do teachers use artefacts, inscriptions, and explanations to represent mathematical concepts and processes in the early years of primary school?
3. What is the rationale behind the teachers' choice of examples, artefacts, inscriptions, and explanations used during lessons?

The first research question focuses on tasks and examples, that are often considered as the basis for mathematics teaching. The second research question examines how the examples in

question 1 are mediated. The third research question looks at the reasons for the teachers' choice of the tasks and examples, as well as their accompanying means of mediation.

1.4 Significance of the study

Studies indicate that early mathematics development predicts learners achievement in later years of school life (Aubrey, Godfrey, & Dahl, 2006; Rittle-Johnson, Fyfe, Hofer, & Farran, 2017; Watts, Duncan, Clements, & Sarama, 2018). However, not many studies have been conducted in the teaching of primary mathematics in Malawi, particularly teaching in the early years (Standards 1–4). It is important to study and understand the teaching before implementing interventions or making suggestions concerning early years mathematics teaching. As such, the study will explore mediation strategies used in the Malawian context and suggest how these can be adapted to different circumstances and contexts. When documented and disseminated, the findings will contribute to pedagogical content knowledge of mathematics teachers for the early years of primary schooling. Documentation and dissemination of the best practices learnt during the study will also inform mathematics teacher educators and policymakers who are responsible for providing support to teachers through continuous professional development.

1.5 Thesis organisation

After this introductory chapter, the second chapter reviews literature related to the effective teaching of mathematics during the early years of primary school. The second chapter also lays out the theoretical framework for the study. Chapter 3 gives the research design and methodology used in the study, explaining the data collection procedures as well as the analytical framework adopted for the study. The fourth chapter presents the findings and discusses them relative to associated theories and existing research. The last chapter outlines the conclusions and implications of the study.

CHAPTER 2

LITERATURE REVIEW AND THEORETICAL FRAMEWORK

2.1 Introduction

This chapter presents a review of literature on some key issues related to the teaching and learning of mathematics during the early years of primary school. The first part discusses the literature on the role of sociocultural factors in the teaching of primary school mathematics in sub-Saharan Africa. This is followed by a general discussion of some empirically supported practices in the teaching of primary mathematics. The last part of the chapter discusses the theoretical framing of the study.

2.2 Mathematics teaching as a cultural activity

As put by Kaur (2017), one of the major lessons from the Third International Mathematics and Science Study (TIMSS) Video Studies is that teaching is a cultural activity. This conclusion is based on the analysis of teaching patterns that reveals close similarities within cultures and huge disparities across cultures. Mosvold (2008) states that teachers share culturally common beliefs that they internalise as they go through their national educational systems. For instance, the cultural beliefs of Dutch mathematics teachers made them use more real-life connections compared to their Japanese counterparts, whose lessons were characterised by a consistent structure (Mosvold, 2008). These variations in teaching practices across cultures may also be reflected in the way teachers represent mathematical ideas using concrete objects, which in turn

affects how the teaching may be described to readers who may not be familiar with that context (Ng, Mosvold, & Fauskanger, 2012).

The complexities and dilemmas faced by mathematics teachers—discussed in Chapter 1—indicate that the teaching practices in mathematics cannot be understood without considering the sociocultural context. Bishop (1988) showed how cultural factors explain the difficulties associated with the teaching and learning of mathematics. He argues that, contrary to popular opinion, mathematical knowledge cannot be considered universal or culture-free; and the more decontextualized and abstracted mathematical ideas are made, the more value-free the subject becomes. This shows that sociocultural considerations are useful when re-thinking about approaches to teaching. This is possibly the reason why Bishop (2017) points to street mathematics as one way of re-thinking about school mathematics in a particular country.

For teachers to be able to harness sociocultural tensions, Tabulawa (2013) proposes the integration of sociocultural consciousness into teacher training and development. He asserts that teacher training that just focuses on the “how” of teaching produces technicians, while training that gives attention to the “why” of teaching develops professional teachers who can “appreciate better the complexity and problematic nature of teaching” (p. 156).

2.3 Sociocultural factors associated with the teaching of mathematics in the early years of primary school in sub-Saharan Africa

There are some cultural and contextual issues that influence the teaching of primary school mathematics in sub-Saharan Africa region. Due to the prevalence of multilingual societies in the region, the issue of language of learning and teaching is commonly discussed in literature.

2.3.1 Contextual problems of teaching and learning mathematics in the early years

Tabulawa (2013) discussed the deep-rooted cultural issues that made some educational interventions fail in sub-Saharan Africa—citing the adoption of learner-centred education as an example. Using the case of Botswana, he argues that quality teaching is not just about learner-centred pedagogy, which does not always work in the sub-Saharan context. Instead of looking at pedagogy in terms of learner centredness or teacher centredness, Tabulawa (2013) advocates for teaching approaches that are supported by the sociological context. Some studies have reported the effects of the sociocultural environment surrounding the school and classroom practices (Davis, Bishop, & Seah, 2015; Hoadley, 2012).

In South Africa, Hoadley (2012) reported research evidence indicating that teachers' social contexts seemed to influence their classroom practices. For instance, teachers located in oral communities seemed to lack a reading culture, resulting in fewer reading opportunities given to their learners in the classroom. However, good practices by teachers from less privileged communities have also yielded positive results. Findings from a study of 15 grade 3 numeracy lessons conducted in six South African schools by Aploon-Zokufa (2013) confirmed the existence of pedagogic strategies that influence low achievement in schools located in poor communities. Results from this study and other similar studies indicated that learner performance was higher when the teacher lets the learners grasp the criteria being used to evaluate their responses throughout the lesson. During such lessons, the teacher explicitly clarified the meaning of concepts and connections between the concepts, giving the learners room for questions and responses.

2.3.2 The teaching of fundamental mathematical concepts during the early years of primary school

Some studies conducted in South Africa have shown that young learners, even those capable of performing arithmetic algorithms, often fail problems requiring the understanding of fundamental concepts like place value, equivalence, or position on a number line, mainly due to deficiencies in the acquisition of number sense (Askew, 2013; Graven, Venkat, Westaway, & Tshesane, 2013). Some issues noted by Graven (2016) and Aploon-Zokufa (2013) include primary school learners' failure to progress from unit counting methods, dependence on manipulative concrete methods of calculating, and failure to recognize incorrectness of the given answer just by considering the place value of the numbers being manipulated.

Another area of concern noted by Aploon-Zokufa (2013) is the lack of connections between a series of examples given by a teacher during a lesson. Anthony and Walshaw (2009) emphasize that new concepts or skills should be introduced in a way that enables learners to make multiple connections with their existing understandings within and across topics, as well as their everyday lives. This can be achieved by giving multiple representations of a mathematical concept. For instance, a teacher may need to make learners realise that adding 7 to 7 is doubling 7, and $\frac{1}{2}$ is 50%. Contextualising mathematical problems to everyday life also makes learners see mathematics as worthwhile. Anthony and Walshaw (2009) caution that improperly chosen contexts can distract learners from the task's mathematical goal. They argue that mathematical tasks should not just be aimed at obtaining the correct answer, but should provide opportunities for learners to struggle with ideas with increasing levels of sophistication.

Another challenge is the existence of learning gaps resulting from the lack of thorough coverage of the learning competencies prescribed in the curriculum for the preceding levels (Graven, 2016). As illustrated by Graven (2016) ignoring gaps in learners' foundational

mathematics knowledge can be compared with compelling builders to lay additional layers of bricks on a structurally weak foundation, where the entire structure falls by the time the ninth layer of bricks is added.

During a study involving teachers of grade 3 literacy lessons, Hoadley (2007) found that learning achievement was higher when the teacher had full control over the selection, sequencing, pacing and evaluation of the knowledge transmitted during the pedagogical discourses. The findings from a study by Aploon-Zokufa (2013) indicated a strong link between the overall performance of schools and the pedagogic practices of its teachers. In Zokufa's study, the strength of a teacher's pedagogic practices was assessed through the extent to which the teacher enacted the pace, sequence and coherence, cognitive demand of the tasks, as well as the nature of feedback during the enactment of the lesson. Hoadley (2012) reported South African studies which noted that lessons progressed at the pace of the slowest learner, negatively affecting coverage of the curriculum.

2.3.3 The language of learning and teaching mathematics in the early years of primary school

The language of learning and teaching is one of the major factors affecting the learning of primary school mathematics in sub-Saharan Africa. Research literature has indicated a strong correlation between the language of learning and teaching and learner achievement (Davis et al., 2015; Graven, 2016). One explanation is that language provides a medium for conveying mathematical concepts to learners as well as assessing learner achievement (Essien, 2018). However, it is not unusual to find mathematics being taught in a language that is not native to some of the learners due to the multilingual composition of many societies. Teaching mathematics in such contexts is very challenging. Hoadley (2012) reports a South African study which found that learners only possessed 1/10 of the required English vocabulary when

switching from vernacular to English as a the language of learning and teaching at grade 5. This made learners lose meaning during lessons, resorting to rote learning, and eventually dropping out of school. Sadly, Anthony and Walshaw (2009) indicated that oftentimes, teachers of mathematics are not aware of the challenges facing students with a different native language and culture.

Despite learners' facing challenges in learning mathematics in a non-native language, the values of the society also seem to have a significant influence. In a study in Ghana relating mathematics pedagogy, language, and sociocultural context, Davis et al. (2015) found that grade 4 learners would willingly forego their understanding of mathematics in preference to the values of the society. Despite failing the word problems given in English during the study, learners in the study still preferred learning mathematics in English saying: "... we don't understand English, that is why we prefer English..." reasoning that "...by so doing we will be learning it [English]" (p. 596). These learners attached more importance to the status that is accorded on one's ability to speak English in the society. On the other hand, learners who preferred learning mathematics in vernacular indicated that "I want to understand the lesson." (p. 595).

In Malawi, Chichewa is used as the language of learning and teaching during the first four classes of primary school (Standards 1–4) (Chilora, Jessee, & Heyman, 2003; Chitera, 2012; Kaphesi, 2003; Kazima, 2008). The use of vernacular languages in the first four classes of primary schools is also done in other sub-Saharan countries such as Kenya and South Africa (Essien, 2018). In a meta-analysis of studies conducted in Kenya, Malawi, and South Africa on the role of language in the teaching of early grade mathematics, Essien (2018) notes a gap in the literature on how teachers and learners in early grades classrooms mediate mathematics terminologies that are not present in their vernacular languages. In an analysis of mother tongue

policies and mathematical terminology in the teaching of mathematics in three countries, Kazima (2008) found that in Malawi mathematical terms were transliterated into Chichewa (by borrowing from English)—compared to the translations done in Nigeria and Tanzania that attempted to carry the meaning behind the mathematical term. For instance, “decimal” was translated to “desimo” and “square” to “sikweya”—which carry no meaning in Chichewa (p. 60).

Furthermore, successful implementation of the use of vernacular in the teaching of mathematics during the early years of primary school requires adequate training of teachers (Chilora et al., 2003; Essien, 2018; Kazima, 2008). A study conducted by Chitera (2012) in Malawi found that mathematics teacher educators in teacher training colleges only have resources developed in Chichewa as the language of learning and teaching in the early years of primary school. However, the study also revealed that mathematics teacher educators in Malawi do not provide pre-service teachers with the opportunities of practising the teaching of mathematics in Chichewa while in college, reasoning that English is the language of learning and teaching in teacher training colleges. Kaphesi (2003) highlighted the conflicts and tensions experienced by practising teachers during classroom teaching while trying to implement the language policy. For instance, the teachers’ guides are in English while the learners’ textbooks are in Chichewa. Besides, the teachers are expected to prepare their lesson plans in English, yet they are expected to teach in vernacular.

In multilingual contexts, teachers can still register success in the teaching of mathematics regardless of the home language differences among the learners. In Malawi, Chilora et al. (2003) conducted a longitudinal study on learners’ performance on mathematical word problems in the early years of primary school in two districts where the language of learning and teaching (Chichewa) was not the home language of most learners as well as their teachers

(Ciyawo). To the surprise of the researchers, learners scored better on mathematics word problems, on average, compared to similar problems that were given in the seemingly simple numeric form (e.g., $23 + 37$)—except those to do with measurement. These findings were contrary to the difficulties that learners face with word problems as reported in other multilingual contexts (Roberts, 2016). There is a possibility that the learners who participated in the study by Chilora et al. (2003) performed very well in the word problems in the language of teaching and learning (Chichewa) because they also use it as their language of play. As such, despite the learners' home language differences, the mathematical problems still made much more sense to them in the common language. This could possibly explain why the difference in performance on word problems between the native and non-native speakers of the language of learning and teaching in Standard 2 was only noticeable during the beginning of the school year, but it was not significant at the end of the year. In the same study, boys outperformed girls on problems related to money probably because of their social differences in exposure to money. This may indicate that the everyday sociocultural context of the learners influences classroom teaching and learning.

2.4 Theoretical framework

Considering that “mathematical knowledge is socially constructed” (Ball, 1993, p. 376) and that mathematics is a “cultural phenomenon” (Bishop, 1988, p. 181), this study is theoretically grounded on Vygotskian sociocultural theory (Kozulin, 2003).

2.4.1 Key concepts

This study conceptualises teaching differently from the practice-based perspective, which looks at teaching as “a plausible conception of professional practice” (Hoover, Mosvold, & Fauskanger, 2014, p. 11). Instead, the study shares Adler and Ronda's (2015) view of the teaching of mathematics as “a sequence of examples and tasks along with the explanatory talk

that follows” (Mosvold & Fauskanger, 2018, p. 210). In addition to verbal talk, teaching mathematics to young learners—which is the focus of this study—entails orchestration of other mediators of learning, such as physical manipulatives (Venkat & Askew, 2018).

Mediation involves meaning making, and is seen through the actions taken by the teacher to make a concept clear to the children in the classroom (Kozulin, 2003). Mediation is often achieved through the use of cultural tools, comprising various learning materials (Kozulin, 2003; Wertsch, 2017). The concept of mediation in the classroom is discussed in section 2.4.3 using the illustration provided by Rezat & Sträßer (2012).

2.4.2 Sociocultural theory

Vygotsky’s sociocultural theory holds that cultural experiences provide the context for learning (Kozulin, 2003). Sfard (2007) also emphasizes that successful learning occurs when there is an agreement between the learner and the teacher. One way of achieving this agreement is through sociocultural history which is shared by the teacher, the learner and the school. As highlighted by Roth and Lee (2007), students live in a network of contradictory activity systems which can only be mediated by their shared ontogenetic histories. They further state that education can be made relevant by moving from objects of traditional school tasks to objects defined within society. In this study, the sociocultural theory is guiding the understanding of the culturally accepted norms of teaching mathematics in the Malawian classroom.

A key aspect of sociocultural theory is that learning is achieved by mediation through the use of cultural tools and a human mediating agent (Kozulin, 2003; Wertsch, 2017). The teacher’s role is to help learners master the cultural tools of mediation to expertise levels where they can use the tools skilfully and flexibly (Wertsch, 2017).

2.4.3 The Mediating Primary Mathematics (MPM) Framework

Several frameworks and lesson observation protocols have been developed to assess the teaching of mathematics. As argued by Charalambous and Praetorius (2018), no single existing framework may capture all aspects of teaching because the frameworks differ in their areas of focus—some focusing on general aspects of teaching (such as teacher-learner interactions) while others focus on content-specific issues (Charalambous & Praetorius, 2018). This study was guided by a content-specific framework that is home-grown for the Sub-Saharan context—the Mediating Primary Mathematics (MPM) framework—shown In Table 2-1.

Table 2-1: MPM Framework (Venkat & Askew, 2018, p. 90).

MEDIATING TASKS/EXAMPLES				
MEDIATING ARTIFACTS				
No artifacts used or artifacts that are problematic/ inappropriate	Unstructured artifacts used in unstructured ways	Structured artifacts used in unstructured ways	Structured artifacts used in structured ways/ unstructured artifacts used in structured ways	
0	1	2	3	
MEDIATING INSCRIPTIONS				
No inscriptions or inscriptions that are problematic/incorrect	Inscriptions that only record tasks or responses	Unstructured inscriptions	Structured inscriptions	
0	1	2	3	
MEDIATING TALK/GESTURES				
Method for generating/ validating solutions	No method or problematic generation/ validation	Singular method/ validation	Localised method/validation	Generalised method/validation
	0	1	2	3
Building mathematical connections	Disconnected and/or incoherent treatment of examples OR Oral recitation with no additional teacher talk	Every example treated from scratch	Talk connects between examples or artifacts/ inscriptions or episodes	Talk makes vertical and horizontal (for multiple) connections between examples/artifacts/ inscriptions/ episodes
	0	1	2	3
Building learning connections: explanations and evaluations of errors/for efficiency/with rationales for choices	Pull back to naïve methods OR No evaluation of offers (correct or incorrect)	Accepts/evaluates offers Accepts learner strategies or offers a strategy OR Notes or questions incorrect offer	Advances or verifies offers. Builds on. Acknowledges of offers a more sophisticated strategy OR Addresses errors/misconceptions through some elaboration e.g., "can it be....?" Would-this be correct, or this? Non example offers	Advances and explains offers. Explains strategic choices for efficiency moves OR Provides rationales in response to learner offers related to common misconceptions OR Provides rationale in anticipation of a common misconception
	0	1	2	3

This framework is grounded on the Vygotskian perspective on the role of sociocultural tools for mediating learning (Venkat & Askew, 2018). The sociocultural perspective also defines the role of the teacher as the sole mediating agent in the classroom (Wertsch, 2017). MPM is also founded on Vygotsky's notion of a scientific discipline as a network of interconnected concepts and procedures (Daniels, 2017; Venkat & Askew, 2018)—as opposed to everyday spontaneous concepts (Daniels, 2017).

The MPM framework was initially called Mathematical Discourse in Instruction—Primary (MDI-P) by the developers in their early writing (Venkat & Askew, 2018). In addition to sociocultural theory, the MPM framework applies variation theory to understand teachers' use of example spaces. On giving attention to the nature of mathematics seen when teaching, this framework has some overlaps with the Mathematical Quality of Instruction (MQI) by Hill et al. (2008) and the Knowledge Quartet (KQ) by Rowland, Turner, Thwaites, and Huckstep (2009). However, the characteristic feature of the MPM framework that differentiates it from MQI and KQ is its tilt towards the mathematical emphasis on structure, relation, and generality, within a sociocultural view of mathematics as a network of scientific concepts (Venkat & Askew, 2018). Just like the MDI framework, which is praised for its focus on clearly delimited observable classroom practice (Mosvold, 2016), the MPM framework also focuses on observable aspects of teaching during a mathematics lesson.

The key features of the MPM framework are discussed by Venkat and Askew (2018). The framework focuses on the nature of the mathematics that is made available to learn and enables a detailed exploration of the quality of primary mathematics teaching. Askew (2019) posits that teaching that does not focus on connections and coherence of mathematical concepts and tasks limits what is made available to learn in a lesson. Venkat and Askew (2018) argue that defining the quality of mathematical teaching by adopting the concept of learner-centeredness

has often been unsuccessful in sub-Saharan Africa. This idea of adopting culturally situated norms of pedagogy as opposed to reforms advocating learner-centred pedagogy largely agrees with the issues extensively discussed by Tabulawa (2013) on the failure of pedagogical reforms in sub-Saharan Africa. The sociocultural theory also assisted in determining the goals in mathematical instruction, as well as the sociocultural materials and practices for mediation.

The MPM framework identified the following four overarching means of mediation, which Venkat and Askew (2018) call strands:

- Tasks and examples
- Artefacts
- Inscriptions
- Talk and gesture

The framework also acts as an analytical tool for assessing the extent to which the mathematics teacher works with each of the four strands of mediation.

Mediation with tasks and examples

The tasks and examples strand in the MPM framework acts as a foundation upon which the teaching is overlaid. An example space in this framework includes all the examples given by the teacher as well as the tasks done by the learners in groups or individually. Venkat and Askew (2018) argues that even though tasks and examples are often considered as objects requiring mediation in literature, they are considered as a mediating strand in the MPM framework. Also, whereas the MDI framework focuses on the use of examples to highlight similarities and contrasts to secondary school learners during lessons, young learners have not yet been inducted extensively into looking for these connections and relationships. As such,

the teacher has to make the relationships and connections explicitly among example spaces to enable learners to notice the mathematical structure and make the necessary generalisations. As such, the framework examines how the tasks and examples strand is mediated by the other three strands (artefacts, inscriptions, and talk and gesture) and applies variation theory (Kullberg, Kempe, & Marton, 2017) to determine relationships within and across example spaces.

Mediation with Artefacts

Askew (2019) defines artefacts as “physical equipment, manipulatives and material objects that have an enduring existence, before and after a lesson” (p. 216). Artefacts serve as the main anchor for mediating early years mathematics. Venkat and Askew (2018) recognise the different roles assumed by artefacts in the teaching of early years mathematics both in their material form and culturally recognised ideal form (that is, as a tool corresponding to a particular purpose). Whereas Venkat and Askew places emphasis on the role of artefacts as tools used by the teacher—who is the main mediating agent in the classroom—others place primary emphasis on the artefacts themselves. Rezat and Sträßer (2012) assert that “any encounter with mathematics is mediated through artefacts” where artefacts include textbooks, tasks and problems, as well as language (p. 644). With this perspective, the mediatory role of artefacts is depicted by Rezat and Sträßer as shown in Figure 2-1 that follows:

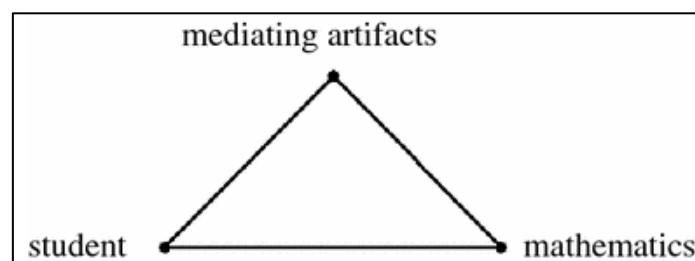


Figure 2-1: The mediatory role of artefacts (Source: Rezat & Sträßer, 2012, p. 644)

The perspective shown in Figure 2-1 represents scenarios where the learning of mathematics is mediated by a tool such as a textbook—or what Venkat and Askew (2018) a “structured artefact” where “the material nature of the artefact presents some possibilities for learners to attend to structure and relations even if the teacher does not make these explicit” (p. 81). While acknowledging the central role of artefacts in the learning of mathematics, the teacher assumes the central role of orchestrating the artefacts in the school setting (Rezat & Sträßer, 2012; Venkat & Askew, 2018). Since the teacher decides what artefacts to use, when to use them, and how to use them, Rezat and Sträßer (2012) proposed a tetrahedral model of the teaching and learning context shown in Figure 2-2.

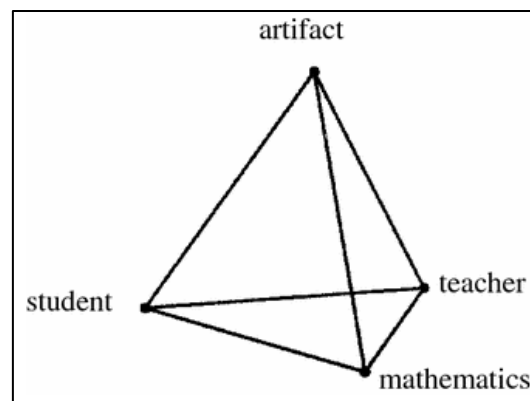


Figure 2-2: The mediatory role of the teacher (Source: Rezat & Sträßer, 2012, p. 645)

As discussed by Rezat & Sträßer (2012) the triangular faces of the tetrahedron model in Figure 2-2 show the various perspectives of the mediational role of artefacts in mathematics education. The basic perspective in Figure 2-1 is also represented in Figure 2-2 by the triangle with vertices *student–artefact–mathematics*. However, since artefacts do not convey mathematical concepts to learners on their own, it is the role of the teacher to transform the use of concrete objects to their ideal form with time. As such, the triangle represented by the vertices *teacher–artefact–student* in Figure 2-2 extends the teacher's role from only being an orchestrator of artefacts to a mediator of learning. Venkat and Askew (2018) show that the teacher’s

orchestration role is achieved by skilfully connecting what the learners already know to what they do not know, that is, linking their everyday mathematics to classroom mathematics. Venkat and Askew emphasize the need to study how the recruited artefacts are used in the teaching of primary mathematics in different contexts. They give an example on the use of the abacus whose functionality can be extended beyond unit counting of numbers, for young learners.

The role of artefacts adopted in this study is represented by the triangle shown by the vertices *teacher–artefact–mathematics* in Figure 2-2. This view signifies the teacher’s role in planning the use of artefacts with respect to the mathematics curriculum (Rezat & Sträßer, 2012). This is necessary because the MPM framework does not just focus on the availability of artefacts in a particular lesson, but how the teacher works with the artefacts—shifting from their physical form to their ideal form where they represent a particular concept or process—which requires thoughtful planning.

Mediation with inscriptions

According to Venkat and Askew (2018), inscriptions are mainly what the teacher writes or draws during the flow of the lesson. This implies that written prewritten charts and diagrams—though containing inscriptions—are regarded as artefacts in the MPM framework because they are not generated during the lesson. This makes inscriptions dynamic and responsive to needs at hand during the lesson. In early years mathematics teaching, it is the teacher’s responsibility to show the representation of actions using physical artefacts as well as their corresponding inscriptions. Using the terminology by Sfard (2008), inscriptions are used in reifying other mathematical processes. Venkat and Askew observe that many teachers dominate the classroom discourse with oral modes of communication as compared to written inscriptions, as noted in learners’ notebooks.

Askew (2019) notes that though some artefacts have inscriptions, such as posters and charts, they cannot be categorised as inscriptions because they are “pre-prepared”. Inscriptions are useful for recording and facilitating moves beyond the immediate presence of physical objects; thus, reifying processes into mathematical objects (Sfard, 2008).

Mediation with talk and gesture

In a typical mathematics lesson, the talk and gesture strand dominates the classroom discourse. Venkat and Askew (2018) explain that the characteristic feature of explanatory talk that separates it from other means of mediation is its self-referential nature. Thus, verbal explanations are used to clarify or build-up on preceding explanations. The teacher uses talk and gesture to make connections among mathematical concepts and to remediate or advance learners existing preconceptions.

The complexity of the talk and gesture strand made Venkat and Askew (2018) further subdivide it into three sub-strands: Talk and gesture for generating solutions to problems, talk and gesture for building mathematical connections, as well as talk and gesture for advancing learning connections.

Talk and gesture for generating solutions to problems

The first sub-strand focuses on teachers’ use talk and gesture in simply finding answers to problems without necessarily considering the explanation to convince the learners. In other words, this is the mediating talk for procedures in arriving at the desired solution.

Talk and gesture for building mathematical connections

The second sub-strand focuses on making connections between mathematical concepts. It is this sub-strand which is closely linked with the conceptual understanding of mathematics. One

technique of making connections involves the application of variation theory to highlight similarities and contrasts during exemplification as a move towards generalisation and abstraction.

Talk and gesture for advancing learning connections

The third sub-strand focuses on advancing learning connections. This requires the teacher not to just accept or reject contributions of learners, but follow through their current level of understanding and build on it. As stated by Ball (1993), an early grade mathematics teacher has to induct learners in advancing their ideas based on plausible arguments rather than relying on the authority of the teacher to validate their responses. As discussed in Chapter 1, when learners recognise that the teacher cannot just be easily persuaded to give an answer, they build confidence in themselves and may start relying on support sought from their peers (Lampert, 1990). Venkat and Askew (2018) noted the absence of responsive evaluation in early years teaching in disadvantaged schools in the South African context, where student offers were simply accepted without discussion of whether they were appropriate.

Significance of the MPM framework in studying early years mathematics teaching

To understand teaching, some classroom-based studies have focused on the pedagogical aspects of teaching. For instance, some have tried to view teaching with respect to the degree of learner engagement in the classroom; hence assess the quality of teaching by determining the extent to which a particular lesson is learner-centred or teacher-centred. However, looking at teaching in this way makes the issue of poor performance paradoxical in the sub-Saharan Africa region (Mhlolo, 2013; Tabulawa, 2013). This view fails to explain the differences in learner attainment among teachers using the same pedagogical styles within the same socio-economic context. This makes it necessary to explore teachers' mediation strategies rather than general teaching styles (Venkat & Askew, 2018).

The MPM framework was considered more fitting for this study because the contextual characteristics of the South African schools that led to the development of the framework share similarities to the schools in Malawi—the context for this study. As discussed by Venkat and Askew (2018), the study schools that led to the development of the framework were impoverished schools where teachers had knowledge gaps in mathematics pedagogy. Rather than just focusing on the deficiencies in the teaching of mathematics identified with frameworks from the developed world, MPM looks at how teachers employ mediational means to the finest detail, making it possible to identify even small changes in practices across lessons (Askew, 2019). The framework also pays particular attention to the mediation of mathematics to young learners, as opposed to more general frameworks on mathematics teaching.

The MPM framework, shown in Table 2-1, has levels that can be used to assess differences in teaching by one teacher at different intervals or among different teachers. This addresses the need for robust constructs for measuring classroom practices noted by Aploon-Zokufa (2013). In this study, however, the numerical levelling aspect of the MPM framework was not adopted because the study did not focus on measuring classroom practices but exploring them. Consequently, only the descriptive levels were adopted as discussed in sections 3.5.6 and 5.6. This led to the modification of the MPM framework presented in section 6.4.

2.4.4 Variation theory

Re-examination of the classroom teaching situation to focus on what is made available to learn, rather than just looking at pedagogical practices, has made some researchers in mathematics education to shift their attention to the success stories of East Asian educational systems, which consistently outperform their Western counterparts during comparative international assessments (Mok, 2017). Analysis of lessons conducted in Asian nations that performed well in Trends in International Mathematics and Science Study (TIMSS) indicated some teachers'

systematic utilisation of variation in the classroom (Jing, Tarmizi, Bakar, & Aralas, 2017; TIMSS, 2015). This has eventually led to the development and adoption of variation theory in the planning, delivery, and evaluation of quality mathematics teaching and learning (Kullberg, Kempe, & Marton, 2017). This is a relatively new learning theory which posits that learning takes place when people's way of seeing and making sense of an object changes. Development of the theory of variation in the teaching of mathematics is mostly attributed to the work of Ference Marton and his co-researchers (Marton & Booth, 1997; Marton, Tsui, Chik, Ko, and & Lo, 2004).

The variation patterns —initially proposed by Marton et al. (2004) — are: *generalisation*, *contrast*, *separation*, and *fusion*. Contrast seeks to help learners discern what something is and what it is not, basing on how it satisfies some criteria. Separation is basically an awareness of part-whole relationship achieved by signifying the invariant parts from a whole. Generalization is inductive in nature and is used in checking the general validity of a separated-out pattern. Fusion aims at bringing out the whole concept by integrating the separated-out critical features of variation together. These four aspects were summarised by Leung (2012) using classification of geometrical plane figures as an example of the object of learning, as shown in Figure 2-3:

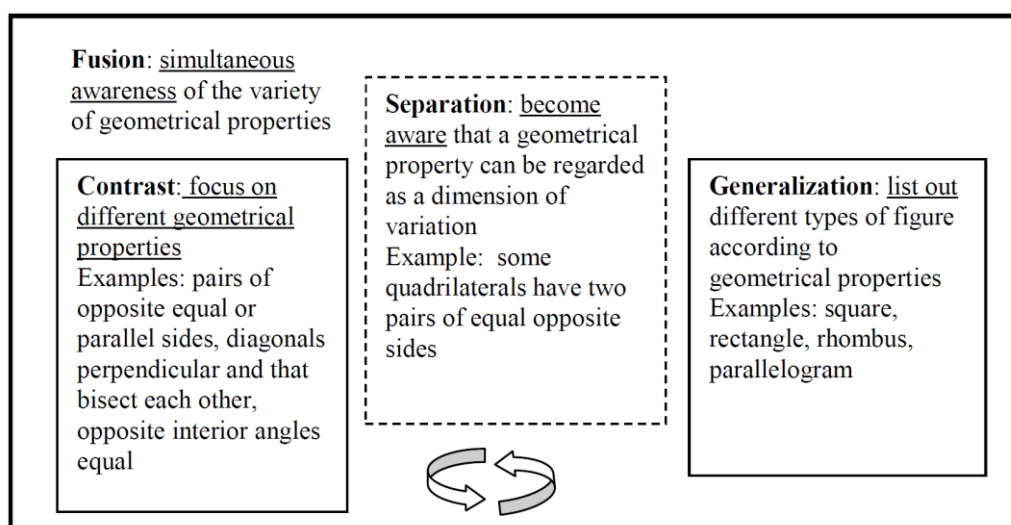


Figure 2-3: Illustration of the four patterns of variation using a geometry example (Source: Leung, 2012, p. 436).

In Figure 2-3, the circular arrows and the dotted rectangle (for *separation*) signify constant interaction between contrast and generalization to achieve the desired awareness among learners.

Leung (2012) proceeds to propose a *discernment unit*, shown in Figure 2-4, as part of a pedagogical process that is driven by the four interacting variation patterns:

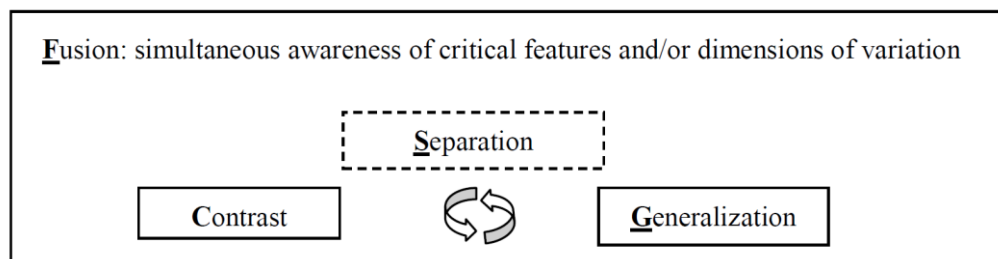


Figure 2-4: Discernment unit (Source: Leung, 2012, p. 437).

In a lesson, there could be several discernment units basing on the awareness that the teacher intends to bring among the learners. Using the example of geometrical properties of shapes presented in Figure 2-3, one discernment unit could focus on classification of plane shapes using their properties, while another unit could focus on the relationships within a particular class or between classes, or other attributes necessary to achieve the intended object of learning.

The way the teacher structures the lesson provides opportunities for learners to perceive the intended object of learning. The quality of mathematics teaching is not only determined by assessing the intended object of learning. What is significantly important is the enacted object learning, which includes the content and the interaction between the learners and the learning tasks (Mok, 2017). Some studies revealed that dimensions of variation are also opened up when students are working individually or collectively (Kullberg et al., 2017). For instance, during a study by Mhlolo (2013), in which he observed a teacher teaching number sequences, he

observed that the questions, comments, and suggestions made by learners in the classroom provided more opportunities of learning than initially intended by the teacher, such as the relationship between sequences and graphs. Some learner's questions invoked more useful varied examples which would have not been given by the teacher. In the end, Mhlolo commended that the lesson gave more opportunities of learning to the learners although advocates of learner-centred education would have out rightly discredited it as teacher centred.

In practice, variation theory involves the use of strategically designed activities that enable learners to discern the object of learning, which is basically the focus of the lesson. As indicated by Kullberg et al (2017) some teachers do not plan their examples used in teaching and neither do they make them a subject of deeper reflection, nor share their choices with colleagues at the school. Variation theory guides teachers in the choice of example spaces on which to base their teaching. As stated by Mhlolo (2013), this requires the teacher to creatively use different variation patterns during the lesson to bring about the desired awareness or discernment among learners. Regardless of the pedagogic practice used by the teacher, the focus of this theory is whether the teacher has made learners discern the object of learning. Thus, after defining the object of learning for a particular lesson, deepening understanding of the object implies acquiring more perspectives on the object, some of which may have previously been taken for granted. When teaching, variation theory capitalizes on patterns of variation such as contrast and generalization to cause learning. Usually, differences between the intended and enacted objects of learning observed after teaching are attributed to the teacher's improper use of exemplification (Kullberg et al., 2017). In this study, variation theory helped in determining the opportunities of learning that were made available by the teachers' selection and sequencing of mediating examples during the lessons. The theory was mainly applied when discussing the teachers' mediating talk for building mathematical connections (see section 5.5.2).

2.5 Chapter summary

This chapter has examined the literature on the sociocultural foundations of teaching mathematics to learners during the early years of primary school. This included a discussion of the theoretical foundations of the MPM framework that is guiding the study. Chapter 3 will examine how the MPM framework was also used as the analytical framework when working with the collected data.

CHAPTER 3

METHODOLOGY

3.1 Introduction

The perspective of any research—from the more general theoretical assumptions to the specific procedures followed—can be described using three major elements: research approach, research design, and research method (Creswell, 2014). As such, this chapter starts with an overview of the methodological assumptions that influenced the design of this study and ends with the tools and techniques used during data collection and analysis.

3.2 Research approach

Depending on the nature of the problem behind a research project, Creswell (2014) states that the research approach can either be qualitative, quantitative, or mixed methods. He further points out that a particular research approach influences the decisions on the plans and procedures followed throughout the research process. This study was driven by the need to explore how teachers in the early years of primary school in Malawi mediate mathematics, as a possible means for identifying ways in which the underachievement of learners that has been reported in the literature (Brombacher, 2011; Jakobsen, Kazima, and Kasoka, 2018) can be improved. In order to explore and understand the nature of teaching in the early years classrooms in Malawi, the qualitative research approach was adopted. The qualitative approach was deemed ideal for this study because of its inherent ability to attend to the contextual richness of the classroom settings where the teaching of mathematics takes place (Yin, 2016).

3.3 Research paradigm

Creswell (2014) described a research paradigm as the worldview that a researcher brings into a study. According to Yin (2016), the researcher's worldview is synonymous with what is also referred to as the epistemological location of a study, guiding the assumptions about the ways of knowing what one knows, and how the research would arrive at its findings and conclusions. Since the study involved human participants, not inanimate objects, there was a need to understand the social influence of their actions while reflecting on the influence of my role as a research instrument (Yin, 2016). To achieve this, meaning making from the findings followed the interpretive research paradigm. The interpretive paradigm was well suited for the chosen qualitative approach mentioned in the preceding section (3.2), the adopted case study design presented in the next section (3.4), and the use of methods that favour natural settings (Chowdhury, 2014) as discussed in section 3.5.

The study was also guided by the sociocultural perspective of teaching and learning in order to understand the mediating of mathematics in the classroom. As such, the study adopted the sociocultural view of the teacher as the main mediating agent in the classroom. The sociocultural paradigm asserts that the style of human mediation cannot be understood without acknowledging the role of available symbolic mediators and, conversely, symbolic tools cannot be understood without a human mediator to facilitate their appropriation by the learner (Kozulin, 2003). Thus, the teacher mediates between the cultural tools for learning mathematics and the learners in the classroom (Venkat & Askew, 2018). Another aspect of the sociocultural paradigm that informed the study is the notion of a scientific discipline as a network of interconnected concepts rather than spontaneous facts (Karpov, 2003). As such, the teaching of mathematics, being a scientific discipline, is expected to be characterised by focusing on interconnected relationships among concepts (Venkat & Askew, 2018). These aspects of the

sociocultural paradigm were instrumental in the selection of methods for data collection and analysis.

3.4 Research design

Research design is defined by Creswell (2014) as the procedures of inquiry or research strategies used in the research. Since the main aim of the study is to understand the complex issue of teaching mathematics to young children, the qualitative case study design was adopted. The case study design allows an in-depth study of a phenomenon in its real world (Yin, 2016). One of the weaknesses often associated with the case study design is lack of generalisability of the results; but Yin (2016) argues that as long as the study was done with rigour, the findings are generalizable to theoretical propositions. He contrasts theoretical generalisations (possible with case studies) with statistical generalizations that are done to populations or universes. As discussed in section 3.6, the inherent weaknesses of the qualitative case study design were addressed by being transparent, methodic, and adhering to evidence (Yin, 2016).

3.4.1 Case characteristics

The study was conducted at a rural school that was purposively selected as a paradigmatic case (Palyst, 2008) exemplifying higher overall performance in poorly resourced rural settings. As argued by Flyvbjerg (2006), a paradigmatic case does not necessarily have specified standards for its selection because it sets the standard. He further argues that a paradigmatic case will ideally have the general characteristics agreeable by the practitioners as an exemplar. Aploon-Zokufa (2013) found a strong link between the overall performance of learners at a school and the pedagogic practices of its teachers—hence the use of learner achievement to identify the case. The assumption was that a well-performing school would provide rich findings on teacher mediation of mathematics whereas a low performing school would likely offer little mediation to observe. Being an exploratory study, the findings from an exemplary school would also

provide insights on best practices that would inform policy and practice related to the teaching of mathematics in the early years of primary school. The limitations of the selected case have been discussed in section 6.5.3.

Since the study focused on teachers of the first four years of primary school, the assumption made was that rural schools have a more stable progression of learners as compared to urban schools. Families in rural areas do not move from one location to another often, making rural schools to have fewer cases of learners transferring from one primary school to another. As such, learners who sit for Primary School Leaving Certificate Examinations (PSLCE) at a rural school are likely to have been at that school from Standard 1. Another assumption for selecting a rural school was that learners from such schools come from nearby villages where the families are of low socioeconomic status, typical of Malawi rural, which is the largest population of Malawi. Hoadley (2007) noted that for middle-class families, the home acts as a second learning space for children, which brings disparities in learning opportunities when such learners are mixed with children from working-class families. The majority of parents from rural schools in Malawi cannot afford learning resources such as textbooks. Furthermore, most of the parents may not have gone beyond primary education, hence they may not have the capacity to offer extra tuition to their children (Kazima, Jakobsen, & Kasoka, 2016). As such, learning (and performance) of learners in Malawian rural schools can solely be attributed to classroom practices of the teachers.

3.4.2 Content delimitation

The study focused on lessons on the addition of numbers. Due to the cross-sectional nature of the study, the focus on one topic of addition enabled gathering data from where the topic was first being introduced in Standard 1 to the last lesson of addition in Standard 4. Addition was chosen considering that additive thinking takes a considerable portion of the early years'

mathematics curriculum. Addition also forms the basis of many other mathematical concepts encountered by learners in later years, including multiplication which is often introduced as repetitive addition. The way a teacher introduces the concept of addition in the early years has a significant effect on the development of additive reasoning in learners (Ekdahl et al., 2018).

3.4.3 Time delimitation

Considering the need to understand the issues surrounding the teaching of mathematics across the four years of primary school within the limited time available for the study, a cross-sectional design was adopted (Saunders, Lewis, & Thornhill, 2009). Data were collected in the 2018/2019 academic year in two major phases. Lesson observations were carried out during the first term (October to November 2018), while follow-up interviews were conducted at the end of each of the last two terms of the school year (April and August 2019).

3.5 Research methods

Creswell (2014) defined research methods as the strategies used for data collection, analysis, and interpretation. This research adopted a multi-method qualitative case study design in which data collection through lesson observation was followed by interviews with teachers and document analysis. This section starts by presenting the strategies used to select the study school. This is followed by a presentation of the professional attributes of the four teachers who participated in the study in Section 3.5.2. Section 3.5.7 provides documentation for part of the data analysis process. It explains the change that occurred after analysing data for the Standard 2 teacher and how the change was done without compromising on the quality of the output from the process.

3.5.1 Study site

Given the pseudonym Zithole, the study school was purposively selected based on the assumptions discussed in section 3.4.1. The school was deemed a paradigmatic case of

exemplary performance in comparison to other schools in the same geographical context. Zithole Primary School achieved a 100% pass rate during PSLCE before the commencement of the study and had a record of higher achievement in a space of five years. The school was identified with assistance from the Ministry of Education's district office responsible for managing Zomba rural primary schools.

During the time of the study, Zithole Primary school had 23 teachers handling a total of 1,481 learners from Standards 1 to 8. The enrolment in the school reduces in the higher primary classes—from 385 in Standard 1 to 58 in Standard 8. This is typical of rural schools in Malawi because of high dropout rates.

One class at each level (Standards 1 to 4) was observed. Since Standards 1 and 2 had two streams, only one of the two streams for each class level was observed. Each class was assigned two teachers sharing all the subjects allocated to that class level. The official minimum age of learners in Standard 1 was 6. However, in all the classrooms, the age of the learners was not homogeneous due to class-level repetitions, as well as due to some learners who got enrolled in Standard 1 past age 6 (Kazima et al., 2016). Figure 3-1 depicts a diagonal view of the Standard 1 class.



Figure 3-1: Part of the Standard 1 class during a lesson (Source: Researcher).

During the study period, the Standard 1 class had the highest number of learners present compared to the other classes observed, with a peak attendance of 170. In Standard 2, the peak attendance was 94. Even though the daily attendance was not consistently recorded in standards 3 and 4, the only recorded attendance for Standard 4 was 150. During an informal interview, the Standard 3 teacher mentioned that her class attendance ranged between 150 and 160.

3.5.2 Study participants

At the school, participating teachers were identified based on the class-levels they were teaching, coupled with their willingness to participate in the study. Where more than one teacher taught mathematics at a class-level, the headteacher made recommendations based on the teachers' work-related performance.

General information about the participants

The study participants were all females. This is typical of primary schools in Malawi to have the early years assigned to female teachers.

Training

All the teachers went through various cohorts of the same 2-year teacher education programme, known as the Initial Primary Teacher Education (IPTE). As stated by Kasoka, Jakobsen, and Kazima (2017), the IPTE programme was tailor-made for the curriculum that the observed teachers were handling. Over the years, teachers in Malawi have been trained through various training programmes that had different structures that possibly influence their competences.

Teaching experience

The teaching experience of the teachers in this study has been presented in Table 3-1. The table includes the number of years that the teacher had been at the study school, the number of years that the teacher had been teaching learners at a particular class-level as well as the number of

years that the teacher had been teaching mathematics regardless of class-level. In Malawi, teachers are normally assigned a particular class-level to teach rather than being assigned a group of learners to move with across class-levels.

Table 3-1: Teaching experience

Class teacher	Years of teaching	Years at Zithole School	Years of teaching the class-level	Years of teaching mathematics
Standard 1	7.5	6.5	4	4
Standard 2	0.9	0.2	0.2	0.9
Standard 3	2.5	2.5	2.5	0
Standard 4	10	1	1	10

From Table 3-1, it can be seen that the overall teaching experience of the teachers varied greatly from 0.2 to 10 years, while the overall number of years spent teaching mathematics at any level varied from 0 to 10 years.

The Standard 1 teacher

The Standard 1 teacher was teaching one of the two Standard 1 classes at Zithole Primary School. As shown in Table 3-1, she had been teaching for seven and a half years. She taught at her first school for 1 year before moving to Zithole Primary School where she had taught for seven years by the time of the study. Upon arrival at Zithole Primary School, she taught other subjects in Standard 5 for about two years before starting to teach Standard 1. This teacher had been teaching mathematics in Standard 1 for four years, up to the time of the study.

The Standard 2 teacher

The Standard 2 teacher had just joined Zithole Primary school after two years from the time she completed her teacher training. Within the two-year period after training, she spent some eight months teaching at a private junior primary school, where she also taught mathematics, after which she resigned to do other non-teaching jobs. By the time she was involved in this

study, she had only been at a public school for two and a half months. As such, her overall teaching experience was the ten and a half months that she taught at both the private and public schools.

The Standard 2 teacher seemed to have some fondness for mathematics such that by the time she was interviewed she was assisting in coaching Standard 5 learners in mathematics during summer break.

The Standard 3 teacher

The case of the Standard 3 teacher was one of the most interesting in this study. It was discovered after she had already been committed to the study that she was not the regular mathematics teacher for the Standard 3 class. She had been teaching at Zithole Primary School for two and a half years (see Table 3-12). During all those years, she had been teaching various groups of learners in Standard 3. Interestingly, she taught all the other Standard 3 subjects over the years except mathematics. Initially, the school had arranged for another teacher to cover for the regular mathematics teacher who was sick, but this teacher was also engaged with her Standard 6 class during the week of observation. The participating teacher was then asked if she would take up the class, though on short notice.

The Standard 4 teacher

The Standard 4 teacher was the most experienced of all the study participants. She had been teaching mathematics during all the 10 years of her teaching career. During her first year of teaching, the teacher also taught mathematics in Standard 1 at her first school. After moving from her first school, she spent 7 years teaching mathematics in Standards 2 and 3 at her second school. Zithole Primary School was her third school. She had taught mathematics for one year in Standard 4 by the time she participated in the study.

3.5.3 Data collection techniques and procedures

Data were collected through lesson observations and follow up interviews with participating teachers, as well as document analysis.

Lesson observations

All the lessons were observed during the last half of the first term of the 2018/2019 academic year in Malawi. The lesson observations were arranged in such a way that they would not disturb the normal routine activities for the school and the observed classes. The number of lessons observed in each class have been presented in Table 3-2.

Table 3-2: Lesson observations across the four classes

Class	Number of lessons	Hours of lesson video
Standard 1	6	7 hours 13 minutes
Standard 2	3	2 hours 28 minutes
Standard 3	4	4 hours 3 minutes
Standard 4	4	4 hours 58 minutes
Total	17	18 hours 42 minutes

The number of lessons observed depended on the teachers' plans recorded in their schemes of work. In some instances, the planned lessons were affected by other unforeseen occurrences at the school, such as sickness of the concerned teachers, and public holidays. The Standard 2 teacher's lessons were also affected by a national health campaign during the week she had scheduled to teach addition. On average

Unstructured interviews

Unstructured post-lesson interviews were done with the teachers to get clarification on some outstanding observations that were made during some of the lessons. These informal interviews helped to establish rapport with the teachers and clarified some of the assumptions they might have had on my role as a researcher, as discussed in section 3.6.1.

In-depth Interviews

Since the in-depth interviews focused on why a participating teacher chose to act in a certain way during the observed lessons, they were conducted after some partial analysis of the data. Due to the time-lapse between the lessons and the interviews, video-stimulated recall was used (Nguyen, McFadden, Tangen, & Beutel, 2013). As discussed by Nguyen et al (2013) video stimulated recall is well suited for probing issues involving decision making in the classroom because it gives participants an opportunity of seeing themselves in action, which helps them recall the thoughts of events as they occurred. On the other hand, the major disadvantage occurs when participants focus on irrelevant details in the video, such as an outfit that was out of place.

In order to maintain rapport with the participants during in-depth interviews, they were asked to decide the most appropriate time to be interviewed, and cooperated with the arrangement. Considering the time required to go through the lesson videos, the teachers indicated that they would only have enough time for the interviews at the end of the school term. As such, two interviews (Standards 2 and 3) were conducted at the end of the second term while the last two (Standards 1 and 4) were conducted at the end of the third term. The order was based on convenience. For instance, it was deemed easier to start by interviewing the Standard 2 teacher, who had the least number of lessons, followed by the Standard 3 teacher who was making arrangements for a transfer to another school. After reviewing the videos, the actual interview session was audio-recorded and transcribed. The teachers were first asked for their consent to record the interviews and they voluntarily accepted. The interview guide has been presented in Appendix 21.

Document analysis

Documents that were checked included the primary school mathematics syllabi for each of the four classes, teachers' guides, lesson plans, as well as schemes and records of work. The issues

that were identified from document analysis have been presented alongside their corresponding findings from lesson observation in Chapter 4. Learners' notebooks were also examined to get some insights on how they grasped the lesson content. Some

3.5.4 Data analysis procedures

The data analysis process started with the initial tasks outlined in Figure 3-2.

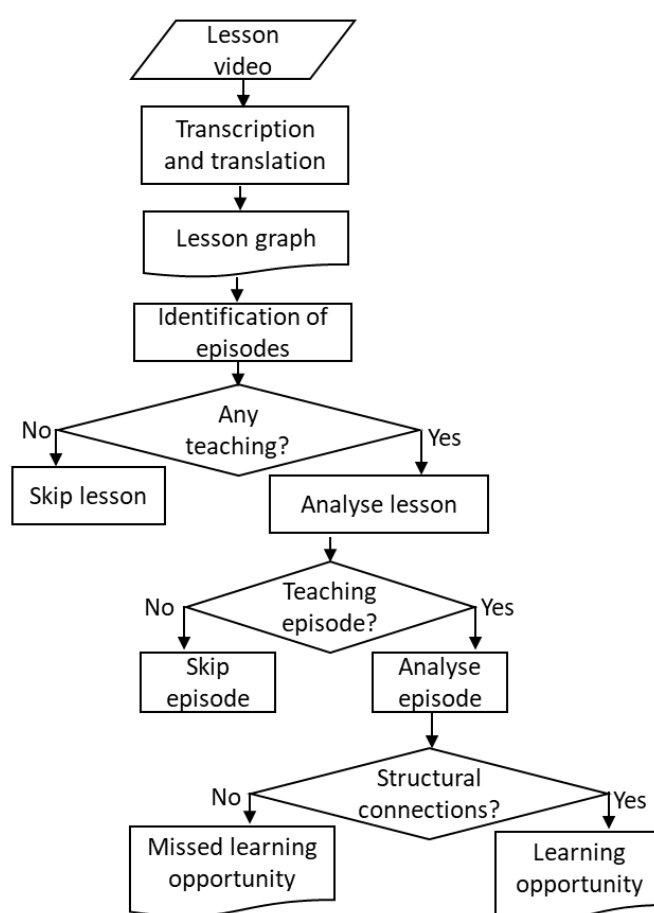


Figure 3-2: General flow of the data analysis process (Source: Researcher).

As shown in Figure 3-2, the first task was the transcription of the lesson videos, which also included a translation from Chichewa to English. This was followed by a summarised description of the lesson using a lesson graph. The next step involved segmenting the lesson into episodes. Before analysing a lesson or an episode, it had to be checked for evidence of new teaching that would provide a rich context for teacher mediation. The last step involved

looking for connections within and between the various means of mediation to determine the extent to which the lesson offered learning opportunities to learners. Each step in Figure 3-2 has been discussed as follows:

Step 1: Lesson transcription and translation

The lesson videos were first transcribed in the language of teaching and learning (Chichewa). Being a native speaker of the language, it was deemed necessary to highlight some of the outstanding observations during the process of transcription. This was done to minimise the possible effects of translation on the findings. Translation was also done to maintain conceptual equivalence of the original statements when translated to English. In some cases, this meant switching between descriptive and technical forms of the same term (Ng et al., 2012). For instance, Chichewa word *kuphatikiza* was conceptually translated as “plus” when it appeared between numbers but in other places it was translated as “addition” or “putting together” (see Table 3-3).

Table 3-3: An example of translation variations for the word “kuphatikiza”

Chichewa phrase	English equivalent
14 <i>kuphatikiza</i> 5	14 plus 5
<i>kuphatikiza nambala</i>	addition of numbers
<i>...kuphatikiza pamodzi...</i>	... adding/putting them together ...

After transcribing the lesson videos, the next step shown in Figure 3-2 involved generation of a descriptive summary of the lesson in the form of a lesson graph.

Step 2: Development of a lesson graph

The first descriptive account of each lesson was a single-page lesson graph (see Appendix 1). The lesson graph showed the major activities of the lesson that were proportionally segmented according to each activity’s duration. As shown in Figure 3-2, the lesson graphs were used as input to the next step of dividing the lesson into episodes. As stated later, the lesson graphs

provided a concise way of determining whether a lesson would be deemed “worthy of further interrogation” (Askew, 2019, p. 216) and hence be coded. An examination of the lesson graphs also helped in determining the basic structure of each lesson, thus partly contributing to the answer for the first research question on the nature of tasks and examples.

Step 3: Segmenting the lesson into episodes

The basic unit for analysing the lesson videos was what Venkat and Askew (2018, p. 81) called an “instructional episode”, which was also referred to as a “mathematical episode” by Adler (2017, p. 130). An episode was marked by a change in the lesson flow, indicating a teacher’s shift in focus to a different aspect of the lesson. This shift often corresponded with changes in tasks or changes in the ways of working, such as from individual work to classwork (Venkat & Askew, 2018). In some instances, the teacher’s talk gave a clue to mark the beginning of a new episode as shown in Utterances 172 and 227 in Excerpt 3-1 that follows.

172. T: Thank you very much! Now I want you to be in your groups and I will give you work to do. ... 227. T: Now I want everyone to take their notebooks and write the exercise...
--

Excerpt 3-1: An example of the beginning of an episode.

The identified episodes for each lesson were documented as *episode summaries*. Appendix 7 shows how Lesson 5 of Standard 1 was segmented into episodes. The lesson graph in Appendix 5 was part of the input to the episode summaries in Appendix 7. It can be noted that some lesson sections shown in the lesson graph turned out to be sub-episodes of a major episode (such as sub-episodes 3.1 and 3.2 in Appendix 7).

After the episodes were identified and summarised, the next step, as shown in Figure 3-2, was to determine whether the identified episodes were analysable with the analytical framework.

Step 4: Determining worthiness of analysis

Each lesson was checked to see if there was something new it was contributing towards answering the research questions in the study. The short descriptions in the lesson graphs and episode summaries simplified the process of checking the extent to which the lesson could be deemed “worthy of further interrogation” using the MPM framework (Askew, 2019, p. 216) and hence be coded.

As exemplified by Askew et al. (2019), episodes that would not be coded in a lesson are those involving “rehearsal or revision of prior learning”, “smoothly run chorused counting episodes”, or “individual seatwork episodes” where mediation by the teacher is minimal or absent (p. 45). Such episodes would not be coded because it could be difficult to establish the need for teacher mediation. This implies that a revision lesson—where learners were just doing recitations, with little or no teacher talk across all episodes, and no incorrect or inefficient learners’ offers—would be deemed not worthy coding using the MPM framework. On the other hand, if incorrect offers or inefficient methods were noted among learners during the rehearsal or revision episodes, it would indicate that the episodes still had a teaching potential and would be coded. Learners’ inefficient methods or incorrect answers are expected to signal the teacher to carry out “responsive moves” (Venkat & Askew, 2018, p. 80) to explain or remediate the noted error (Askew, 2019). In other words, incorrect offers from learners during a lesson signify what Muir (2008) referred to as “teachable moments” (p. 362).

During the study, for instance, the teachers initiated number songs at the beginning of the lesson. The songs were not coded because evidence of teacher mediation could not be established. Such songs could only be coded if there was a mediational affordance related to the lesson, such as where the teacher refers to the number song when explaining a concept. Similarly, marking sessions were coded depending on the teacher’s actions during that period.

Some marking episodes were analysed because the teacher actively intervened providing the required supporting explanations to any observations made as the learners solved the problems. During some sessions, the teacher would mark quietly, without any observable mediation, hence were not coded.

3.5.5 The initial coding scheme

Codes were developed deductively from the MPM theoretical framework with its analytical tool for lesson analysis. This section discusses the MPM coding scheme in its entirety. This is followed by a discussion of the challenges associated with the numerical levelling associated with the framework in section 3.5.6. The actual implementation of coding has been presented in section 3.5.7.

Coding for mediating tasks and examples

The term *task* usually means a mathematical problem that learners work with (Mosvold, 2016). Adler and Ronda (2015) related tasks with examples by extending the definition of a task to mean “what learners are asked to do with the various examples presented” (p. 241). As such, in this study, a task contained one or several examples. The set of examples within a mathematical task also provides a *space* for exploring horizontal connections among them or vertical connections with other example spaces (Askew, 2019) through the application of variation theory (Kullberg et al., 2017; Venkat & Askew, 2018).

Since young learners may not have fully developed the capability of deducing the variant and invariant aspects of example spaces by themselves, Venkat and Askew (2018) indicate that more opportunities to learn are realized when the teaching draws attention to such aspects through talk and gesture. Hence, examples and tasks are not analysed independently in MPM, but just listed when they occur in an episode during the lesson. Table 3-4 summarises the coding of examples and tasks in a lesson.

Table 3-4: Coding scheme for examples and tasks. Adapted from Venkat and Askew (2018, p. 90)

Description	Coding
Examples and tasks	Only coded for occurrence, but not for an independent analysis

In the MPM framework, an example space includes individual and group work given to the learners.

Identification of the object of learning

In a typical lesson, all the examples and tasks are directed towards the achievement of a goal, which is the object of learning. The object of learning can also be expressed as “what learners are expected to know and be able to do” after the lesson (Adler & Ronda, 2015, p. 238). Oftentimes, the object was explicitly mentioned by the teacher as shown in Excerpt 3-2, mostly presented as the topic of the week.

5. T: Thank you very much. Today, we will start learning the addition of numbers. We will start learning the addition of numbers. What will we learn?
 6. C: Addition of numbers!

Excerpt 3-2: Object of learning for Lesson 1 of Standard 1.

The object of learning was also implicitly deduced from the given tasks and examples. The deduced object often indicated a narrower lesson goal than the one stated by the teacher (see Table 3-5).

Table 3-5: Stated object of learning versus observed object of learning in Standard 2 lessons

Lesson	The stated object of learning	The observed object of learning
1	Addition of numbers up to 20	Number bonds of 10
2	Addition of numbers up to 20	Number bonds of 12
3	Addition of numbers up to 50	Place value addition

Determining the object of learning for the lesson made it easier to understand the teacher's actions during the lesson.

Coding for mediating artefacts

As stated by Venkat and Askew (2018), artefacts refer to the tangible resources prepared by the teacher before the lesson and may remain in existence after the lesson. The coding for artefacts has been summarised in Table 3-6.

Table 3-6: Coding scheme for mediating artefacts. Adapted from Venkat and Askew (2018, p. 90)

Level	Indicators for the usage of artefacts
0	No artefacts used, problematic artefacts, or inappropriate artefacts
1	Unstructured artefacts used in unstructured ways
2	Structured artefacts used in unstructured ways
3	Structured artefacts used in structured ways or unstructured artefacts used in structured ways

Coding for mediating inscriptions

Inscriptions are what the teacher writes during the flow of the lesson. Venkat and Askew (2018), as well as Askew (2019), differentiated artefacts and inscriptions based on whether they are pre-made and brought into the classroom, as well as their permanence after the lesson. Inscriptions are temporary in nature. This implies that charts and cards prepared before the lesson, though containing inscriptions, would be regarded as artefacts in the MPM framework. Table 3-7 shows a summary of the coding for mediating inscriptions in the MPM framework.

Table 3-7: Coding scheme for mediating inscriptions. Adapted from Venkat and Askew (2018, p. 90)

Level	Indicators for the usage of inscriptions
0	No inscriptions, problematic inscriptions, or incorrect inscriptions
1	Inscriptions that only record tasks or responses
2	Unstructured inscriptions
3	Structured inscriptions

Regarding the usage of inscriptions in Table 3-7, the focus was on how the teacher systematically presented the inscriptions to enable learners to discern mathematical properties and relationships. Structured inscriptions were those that were systematically ordered to bring attention to the conceptual properties of numbers and their operations. Unstructured inscriptions, on the other hand, refer to those that were randomly presented with no attention to mathematical structure and connections within and between examples. In some cases, the inscriptions would only record the tasks and examples given by the teacher or the responses from learners.

Coding for mediating talk and gesture

The MPM framework subdivided mediating talk and gesture into three strands: Mediating talk and gesture for providing methods for generating solutions, building mathematical connections, as well as advancing learning connections (Venkat & Askew, 2018).

Talk and gesture for generating solutions

This sub-strand focused on how the teacher arrived at solutions for the problems being worked out during the lesson. It also includes the methods used by the teacher for validating solutions offered by learners. The coding for talk and gesture for generating solutions has been presented in Table 3-8.

Table 3-8: Coding scheme for mediating talk and gesture for providing methods for generating solutions. Adapted from Venkat and Askew(2018, p. 90)

Level	Indicators for generating solutions to problems
0	No method or problematic method for generation or validation of solutions
1	Singular method for generation or validation of solutions
2	Localised method for generation or validation of solutions
3	Generalised method for generation or validation of solutions

Talk and gesture for building mathematical connections

As highlighted by Venkat and Askew (2018), variation theory posits that more learning opportunities are achieved when the teacher looks for structural similarities and contrast to show the connections between sets of examples (example space). The coding for talk and gesture that builds mathematical connections has been summarised in Table 3-9 that follows.

Table 3-9: Coding scheme for building mathematical connections. Adapted from Venkat and Askew (2018, p. 90)

Level	Indicators for building mathematical connections
0	Disconnected examples. Incoherent treatment of examples. Oral recitation of examples with no additional teacher talk
1	Every example treated from scratch
2	Talk connects between examples, artefacts, inscriptions, or episodes
3	Talk makes vertical and horizontal multiple connections between examples, artefacts, inscriptions, or episodes

Talk and gesture for advancing learning connections

This aspect of mediating talk focused on teaching that was attentive to learners’ responses and addressed them according to their understanding needs. This was where the teacher was prompted to make “responsive moves” (Venkat & Askew, 2018, p. 80) to take advantage of “teachable moments” (Muir, 2008, p. 362) that came up during the lesson. During a lesson, the teacher’s explanatory effort would need to be directed on the least understood material or on

material that would make subsequent lessons be least understood to the learners (Muir, 2008). The coding for teacher's mediating talk for advancing learning connections has been summarised in Table 3-10.

Table 3-10: Coding scheme for advancing learning connections. Adapted from Venkat and Askew (2018, p. 90)

Level	Indicators for advancing learning connections
0	Pull back to naïve methods or no evaluation of offers (correct or incorrect)
1	Accepts or evaluates offers, accepts learner strategies, offers a strategy, notes or questions incorrect offer
2	Advances or verifies offers, builds on, acknowledges or offers a more sophisticated strategy, addresses errors and misconceptions through some elaboration e.g., "can it be....?" Would this be correct, or this? Non-example offers
3	Advances and explains offers, explains strategic choices for efficiency moves, provides rationales in response to learner offers related to common misconceptions, provides rationale in anticipation of a common misconception

As shown in Table 3-10, the teacher's talk would need to examine learners' explanations, checking them for errors and efficiency. Desirable teacher's talk builds on the novel, but efficient strategies given by learners. This can be contrasted with teaching that pulls back learners from their efficient strategies to the teacher's naïve strategies. An example of pull-back teaching could be where a teacher may request learners to do concrete counting, yet the learners could mentally generate solutions.

Overall application of the MPM coding scheme

Table 3-11 contains the coding of Lesson 1 of Standard 2 that was parsed into 5 episodes and analysed using the scheme discussed in section 3.5.5. In the table, the talk and gesture strand shows its three contracted sub-strands: Generating solutions [GS], mathematical connections [MC], and learning connections [LC].

Table 3-11: Overall coding for Lesson 1 of Standard 2. Adapted from Venkat and Askew (2018, p. 90)

Episode	Example space	Artefacts		Inscriptions		Talk and gesture		
		Type	Usage	Type	Usage	GS	MC	LC
1	$12 + 5$	Counters	1	$12 + 5 =$ written vertically	0	1	1	2
2	$1 + 9, 6 + 4$	Counters	1	$1 + 9 =$ and $6 + 4 =$ written on top of each other	2	1	2	2
3	$2 + 8, 5 + 5$	Counters	1	$2 + 8 =$ and $5 + 5 =$ written side by side	3	1	3	2
4	$8 + 2, 3 + 7, 5 + 5$	Counters and Prewritten papers	2	No new inscriptions by the teacher	-	1	3	2
5	$11 + 6, 14 + 5, 15 + 3$	Counters, fingers and toes	1	$11 + 6 =$, $14 + 5 =$, $15 + 3 =$ presented on the chalkboard	1	1	1	2

The issues related to the usage of the MPM's coding scheme in its entirety have been discussed in the following section.

3.5.6 Issues with the adoption of the MPM coding scheme

Most of the challenges that were faced with the use of the MPM coding scheme were related to the numerical weighting of the teacher's mediational moves.

Issues with weighting the extent of mediation

The coding scheme of the MPM framework assigns numerical values, called levels, for measuring the quality of mediation (Venkat & Askew, 2018). This feature is useful for a study that intends to measure differences in teaching for a particular teacher across two time periods or differences between two teachers within the same period. Since this study was not aimed at comparing teaching, the assignment of numerical scores as presented in the original framework

was not adopted. Also, if scoring was done, each score would necessitate further justifications for the following reasons:

- The levels suggest a continuum. As such, two independent researchers may differ on whether they perceive the observed action as being closer to, say, level 2 or level 3. To mitigate this possibility, scoring in the MPM framework is discussed and agreed by pairs of the research team (Askew, 2019). Since this study was carried out by one researcher, such a discussion would not be possible.
- The scores suggest an ordinal scale of measurement. But since this was not explicitly stated, questions may arise as to whether the move from level 1 to level 2 is equivalent to the move from level 2 to level 3.
- For each score, there would be a need to justify whether the 3 of inscriptions has the same numerical significance as a 3 of artefacts.
- Due to uncertainty on the scale of measurement, level 0 may suggest the absence of mediation, as such one would expect mediation that “pull-back to naïve methods” (Venkat & Askew, 2018, p. 90) to have a score of less than 0, as it can be considered technically lower than the absence of mediation.

The above questions would arise considering that Askew et al. (2019, p. 45) quantitatively used mediational scores to produce “summed”, “fraction[al]” and “averaged” scores for the use of particular means of mediation.

Adoption of descriptive coding

Due to the reasons discussed above, this study used the MPM analytical framework to describe the extent of mediation noted among the participating teachers, but not comparing them through numerical scores assigned to their quality of teaching.

3.5.7 Changes in the implementation of data analysis

Data analysis was first done with the Standard 2 teacher, followed by teachers of Standards 1, 3, and 4, in that order. This section points out the adjustments that were made in the analysis of the data for the last three teachers in comparison to the analysis previously done with the first teacher

Reasons for changing the approaches

Data analysis started with the Standard 2 teacher because the lessons were fewer and relatively shorter. This made it easier to work with the data analysis tools and techniques used during the pilot study.

When analysis of the lessons for the Standard 1 teacher started, it was deemed necessary to adjust the tools and techniques used for data analysis because this teacher had six lessons, most of which exceeded 1 hour (see Table 3-2). Thus, it was anticipated that the volume of data to be generated would require more time to handle. This made it necessary to reconsider the strategies for the management and retrieval of the analysed data for the Standard 1 teacher and the subsequent teachers whose lessons also averaged over an hour. The next sub-section starts with a comparison of how data analysis was done for the Standard 2 teacher using Microsoft Word (MS Word) and how it was done with the rest of the teachers using ATLAS.ti qualitative data analysis software. This will be followed by a detailed discussion of the implementation of data coding and retrieval using ATLAS.ti.

Comparison of the tasks done during the analysis using MS Word and ATLAS.ti.

Technically, the volume and quality of data generated after migrating from MS Word to ATLAS.ti remained the same because the same codes were used. The key difference was the speed of coding and retrieval because the separate MS Word transcripts for one class were managed as one entity in ATLAS.ti (called a *document group*), thus easing data coding and

retrieval. Table 3-12 that follows shows how similar tasks were done during the analysis of the data for the Standard 2 teacher and the other three teachers.

Table 3-12: Comparison of analysis tasks between the Standard 2 teacher and the teachers for Standards 1, 3, and 4

Analysis task	Standard 2 teacher	Teachers for Standards 1, 3, and 4
Lesson overview	Lesson graph	Lesson graph
Description of lesson tasks	Episode summaries	Episode summaries
Data management	MS Word	ATLAS.ti
Coding of utterances	References to utterance numbers	Tagging of utterances.
Overall summary of coded utterances	Compiled MS Word report	Query-generated reports in MS Excel or MS Word

The actual processes that were done on the handling of data have been presented in the next sub-section.

Coding implementation in MS Word relative to ATLAS.ti

The migration from MS Word to ATLAS.ti mainly targeted the mechanical tasks without negatively affecting the methodological process. To illustrate how the implementation did not affect the desired goals of the analysis process, this section shows a snippet of the analysis that was done for a Standard 2 lesson transcript in MS Word (in Figure 3-3) and how the same process was implemented for a Standard 1 transcript in ATLAS.ti.

Coding in MS Word

Initially, coding was done through comments that were made along a column that ran alongside the analysed transcript in MS Word. Each comment had hyperlinked references to the utterances containing the reported observation, such as {62} and {80} appearing in Figure 3-3. The comments were categorised based on the reported means of mediation (mediating tasks and examples, artefacts, inscriptions, talk and gesture).

64. T: Don't shout! Madam! Madam! Alright.	textbook to check examples to use {62}.
65. C: Yes!	
66. T: Eeh! You! [Pointing to a learner]	
67. L7: [Teacher points to each part of the addition statement with a piece of chalkboard ruler] 1 plus 9 equals!	Worked with addends having a sum of 10 {80}-{81}
68. T: She says 1 plus 9 equals. She is correct, alright?	<u>Mediating artefacts</u>
69. C: Yes!	Used counters {72}, {75}, {79}
70. T: Let us clap hands for her!	
71. C: [Clap hands]/	Counted with learners {72}, {75}.
72. T: 1 plus 9 equals. [Picks counters from teachers table] Let us count together! 1! [Pushes one piece of the counters]	Used count-all strategy to find the sum {78}.
73. C: 1!	<u>Mediating inscriptions</u>
74. T: We should add 9!	Asked learners to read chalkboard inscriptions before finding solutions {62}, {82}.
75. C+T: [Teacher pushing each piece of the counters] 1,2,3,4,5,6,7,8,9!	

Figure 3-3: Snippet of the analysis for Lesson 1 of Standard 2 (Source: Researcher).

As shown in Figure 3-3, text colours were systematically used to ease the identification of comments and codes for the different means of mediation. The task of tracking colours and hyperlinking utterances when coding was greatly reduced after migrating to ATLAS.ti.

Coding in ATLAS.ti

Even though the use of colouring was possible in ATLAS.ti, it became necessary to develop a recognisable system for naming codes to ease coding and retrieval. For example, the usage of inscriptions described as “Asked learners to read chalkboard inscriptions before finding the solutions” in Figure 3-8 was assigned the equivalent code named “Inscriptions:Reading” in ATLAS.ti (see Table 3-13).

Some codes were created based on the MPM framework while other open codes were created based on recurring themes observed in the data. To illustrate the coding scheme that was used,

Table 3-13 lists some of the codes that were used for tagging utterances related to inscriptions in the analysed transcripts.

Table 3-13: Systematic naming of codes for inscriptions in ATLAS.ti

No.	Code	Description
1.	Inscriptions:Learners	Inscriptions written by learners
2.	Inscriptions:Nature	Nature of inscriptions
3.	Inscriptions:Reading	Reading the given inscriptions
4.	Inscriptions:Teacher	Inscriptions written by the teacher

The codes in Table 3-13 were used for coding text. Some codes were also developed for tagging of images that were embedded in the transcripts based on the themes associated with the images. Table 3-14 lists some of the codes that were used for tagging images related to inscriptions. Most of these images were generated from the screenshots of the recorded videos while some were captured using a camera during the lesson.

Table 3-14: Codes for images

No.	Code	Description
1.	#image:inscriptions_learners	Inscriptions written by learners
2.	#image:inscriptions_nature	Nature of inscriptions
3.	#image:inscriptions_teacher	Inscriptions written by the teacher
4.	#image:inscriptions_use	Teacher's use of inscriptions

In order not to compromise the trustworthiness of the findings from the many generated codes, related codes in the codebook were grouped. Queries were mostly run based on particular code groups rather than individual codes (see Figure 3-4).

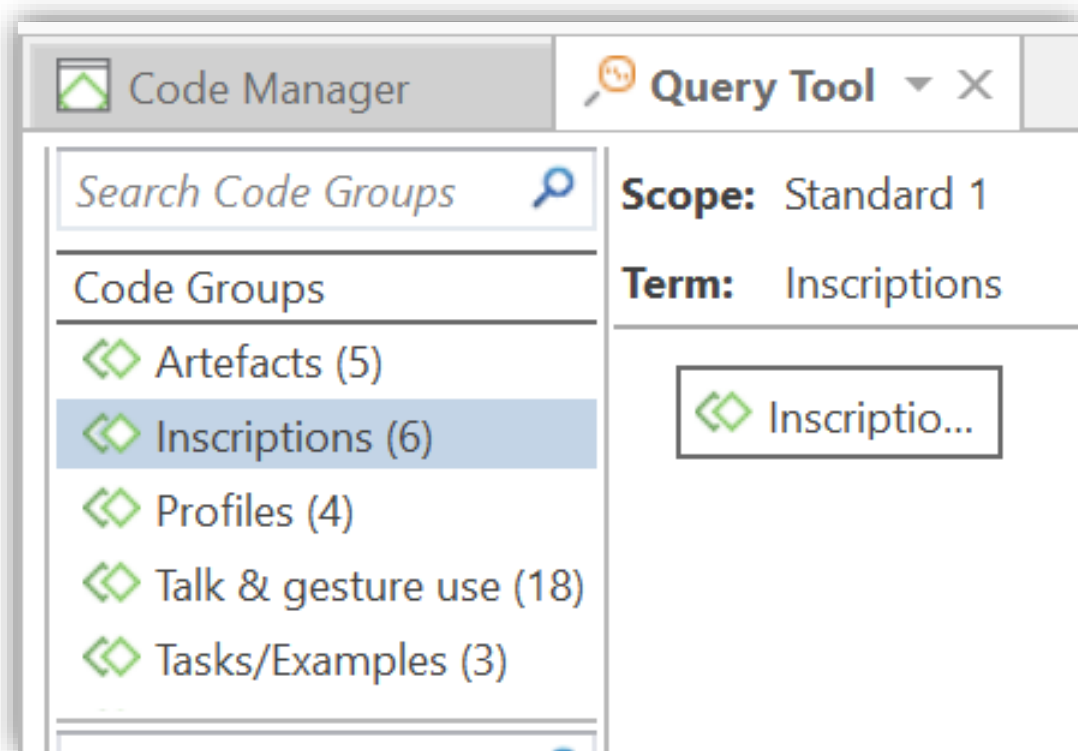


Figure 3-4: Screenshot showing the use of code groups when running queries in ATLAS.ti (Source: Researcher).

In ATLAS.ti, the process that is shown in Figure 3-3 (using MS Word) was done by tagging the transcripts with the codes like those presented in Table 3-13 for each means of mediation. Figure 3-5 shows a screenshot of the coding applied to Utterance 558 in ATLAS.ti for the transcript of Standard 1 Lesson 1.

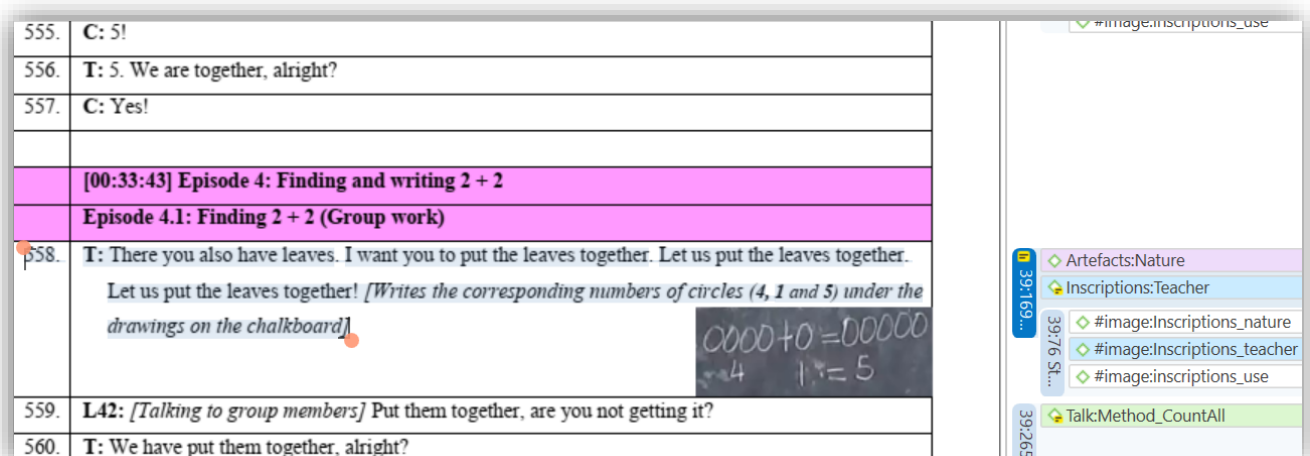


Figure 3-5: Coding of utterances of Standard 1 Lesson 1 in ATLAS.ti (Source: Researcher).

Comments that were done in MS Word (like those shown in Figure 3-3 and Figure 3-7) were also implemented in ATLAS.ti for some remarkable utterances. Comments were added to the utterances as shown on the right-hand pane of Figure 3-6 that follows. The comment shown in Figure 3-6 was applied to Utterance 558 together with the codes shown in Figure 3-5.

556.	T: 5. We are together, alright?
557.	C: Yes!
[00:33:43] Episode 4: Finding and writing 2 + 2	
Episode 4.1: Finding 2 + 2 (Group work)	
558.	T: There you also have leaves. I want you to put the leaves together. Let us put the leaves together. Let us put the leaves together! <i>[Writes the corresponding numbers of circles (4, 1 and 5) under the drawings on the chalkboard]</i>
559.	L42: <i>[Talking to group members]</i> Put them together, are you not getting it?
560.	T: We have put them together, alright?
561.	C: Yes!

AutoFacto: Notebook

The teacher represented stones with circles drawn on the chalkboard followed by their corresponding numbers below the drawings.

Go to Quotation

Figure 3-6: Commenting on utterances in ATLAS.ti (Source: Researcher).

Retrieval of coded utterances in MS Word

When using MS Word, the task of tracking codes for retrieval started with the compilation of summaries of coded utterances at the end of each episode. Figure 3-7 shows a snippet of a summary that was made at the end of Episode 3 of Standard 2 Lesson 3.

Mediating artefacts

The teacher asked a learner to use two place-value boxes to find $28 + 11$ as she demonstrated in Episode 2 {283}.



During verification of the learner's answer, the teacher worked with the bundles and sticks representing the numbers in the place-value boxes {259}, {283}, {285}, {291}, {299}, {307}, {314}, {321}-{331}, {335}-{337}.

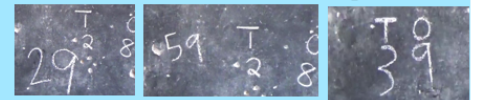


Mediating inscriptions

The teacher wrote 28 and 11 on separate sections of the chalkboard and asked learners to read the written numbers {261}, {269}, {323}, {354}.



The teacher also asked learners to write the answer (39) on the chalkboard, even when that required the learners making several attempts {337}-{368}.



The teacher rewrote the problem in column addition format

Figure 3-7: Snippet of a summary made at the end of Episode 3 during the analysis of Standard 2 Lesson 3 using MS Word (Source: Researcher).

The analysis of a lesson ended with an overall summary of the utterances associated with a particular observation. Figure 3-8 shows part of the overall summary on the use of artefacts during the first lesson of Standard 2.

episode 4, learners also worked with pre-written papers which had three problems for them to solve in their groups{174}. Occurrences of usage of artefacts during the lesson have been tabulated below:					
Usage	Incidents in each episode				
	Episode 1	Episode 2	Episode 3	Episode 4	Episode 5
Pushing counters one-by-one while mentioning numbers.	{19}, {23}, {25}	{72}, {75}, {79}, {103}, {107}, {110}	{143}, {146}, {148}, {150}, {155}, {161}, {163}, {165}	{198}, {200}, {206}, {208}, {210}, {214}	{260}, {261}, {309}, {311}, {313}
Counting all to get the sum.	{24}	{78}, {109}	{154}, {164}	{201}, {209}, {217}	{287}, {288}, {312}
Ensuring that each learner was counting using counters.		{99}-{103}	{137}-{138}, {149}	{172}, {178}	{256}, {279}-{281}
Emphasis on similarity				{183}-{193}	

Figure 3-8: Snippet of overall summary on the use of artefacts during the analysis of Standard 2 Lesson 1 using MS Word (Source: Researcher).

Retrieval of coded utterances in ATLAS.ti

The source utterances, such as those marked {174} and {19} in Figure 3-8, were easily accessible in ATLAS.ti. Utterances related to a particular code could be retrieved by running a query based on the desired parameters. For instance, it was possible to run a query retrieving all utterances on the teacher's use of inscriptions in one lesson or a range of lessons. Outputs from the queries could be viewed within ATLAS.ti or exported as MS Excel spreadsheets as well as MS Word reports. The ATLAS.ti screenshot in Figure 3-9 shows the query for all the six codes for inscriptions across all the Standard 1 transcripts. One of the outputs includes Utterance 558 coded earlier in Figure 3-5 (displayed under "Quotations" in Figure 3-9).

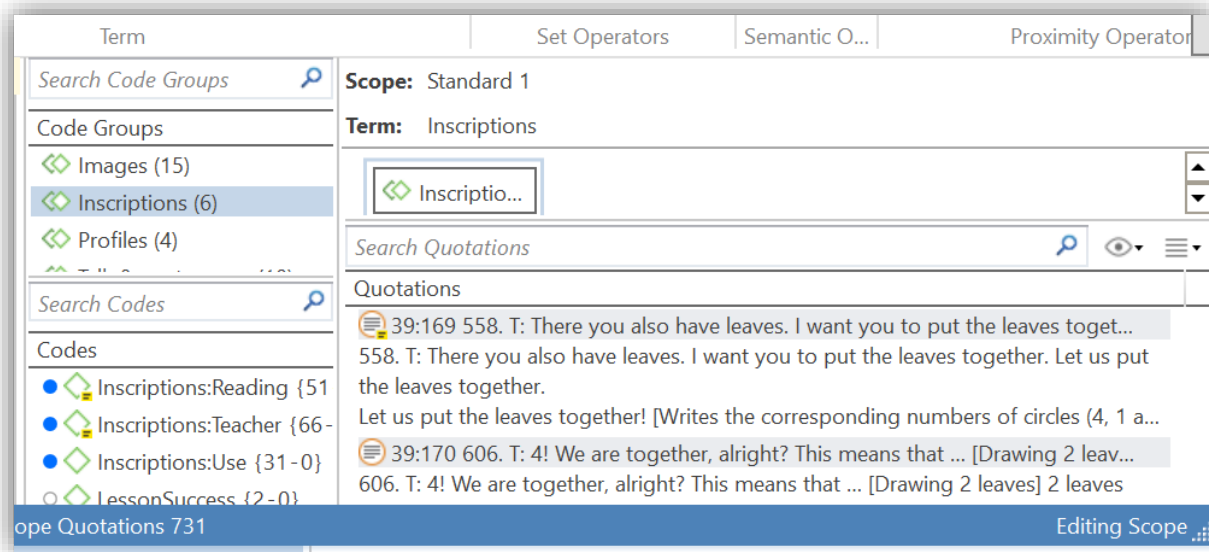


Figure 3-9: Screenshot of an ATLAS.ti query for use of inscriptions in Standard 1 (Source: Researcher).

A sample output from an exported MS Excel spreadsheet has been presented in Figure 3-10. The Excel screenshot in Figure 3-10 displays a report from the query run in Figure 3-9. The output was scrolled to display Utterance 558 with its associated codes and comments previously shown in Figure 3-5 and Figure 3-6.

	C	D	E	F
	Document Group	Quotation Content	Comment	Codes
1	Study Data Standard 1	558. T: There you also have leaves. I want you to put the leaves together. Let us put the leaves together. Let us put the leaves together! [Writes the corresponding numbers of circles (4, 1 and 5) under the drawings on the chalkboard]	The teacher represented stones with circles drawn on the chalkboard followed by their	Artefacts:Nature Inscriptions:Teacher
17	Study Data Standard 1	606. T: 4! We are together, alright? This means that ... [Drawing 2 leaves] 2 leaves plus? [Writes +] plus so many leaves? 607. C: 2!		Inscriptions:Teacher

The screenshot shows an Excel spreadsheet with the following data:

	C	D	E	F
	Document Group	Quotation Content	Comment	Codes
1	Study Data Standard 1	558. T: There you also have leaves. I want you to put the leaves together. Let us put the leaves together. Let us put the leaves together! [Writes the corresponding numbers of circles (4, 1 and 5) under the drawings on the chalkboard]	The teacher represented stones with circles drawn on the chalkboard followed by their	Artefacts:Nature Inscriptions:Teacher
17	Study Data Standard 1	606. T: 4! We are together, alright? This means that ... [Drawing 2 leaves] 2 leaves plus? [Writes +] plus so many leaves? 607. C: 2!		Inscriptions:Teacher

Figure 3-10: Screenshot of an Excel report on teacher's use of inscriptions (Source: Researcher).

The process illustrated in the preceding sections for lesson transcripts was also used for coding interview transcripts and running queries to retrieve the required reports.

Inductive coding

In addition to the deductive codes developed from the MPM framework, some inductive codes were developed based on the observations that were prevalent in the transcripts. One way of noting prevalent issues in one transcript or a set of transcripts was through the use of word clouds as well as frequency tables for all the words found in the transcript(s). This process was usually the first to be done when a new transcript was uploaded. Figure 3-11 depicts a word cloud containing all the words that exceeded 100 occurrences across all the Standard 1 transcripts.

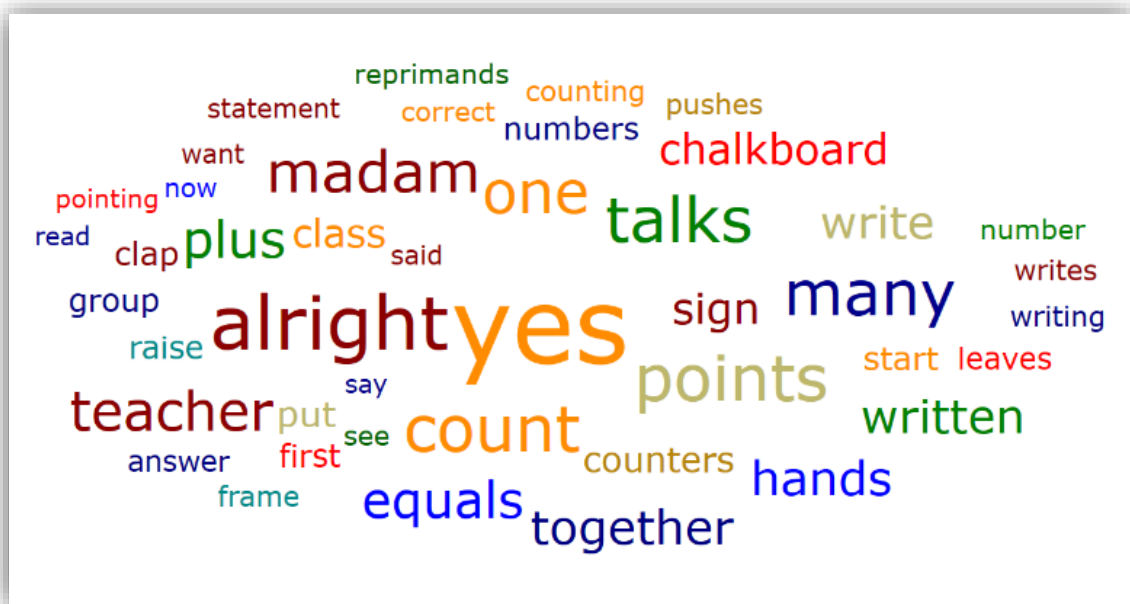


Figure 3-11: Word cloud showing words that exceeded 100 occurrences across all the Standard 1 transcripts (Source: Researcher).

In addition to using them for generating new codes, word clouds also provided a quicker way of coding utterances that contained a particular word. For instance, the word “reprimands”

appearing at the top in Figure 3-11, had 119 occurrences across the six Standard 1 lessons, giving a picture of the Standard 1 class-profile that would possibly be left out when coding deductively using the adopted analytical tool for the study. It should be noted, however, that ATLAS.ti only performs a basic word count and does not provide content-based parameters for tabulating the frequency counts for specific sources of text indicated within the document. For example, the word “reprimands” appearing at the top in Figure 3-11 did not come directly from the teacher, but from the comments made in the transcript for the actions made by the teacher and the learners. As such, sources of a word appearing in a word cloud or word list were checked before making any inferences from the displayed occurrences.

3.5.8 Summary of data collection and analysis tools

The formats of the data that was generated during the processes described in sections 3.5.4 and 3.5.7 have been listed in Table 3-15.

Table 3-15: Summary of outputs generated during data collection and analysis

Research activity	Output data format
Lesson observation	Recorded lesson videos, field notes, photographs
Unstructured post-lesson interviews	Fieldnotes.
Video stimulated recall interviews	Recorded audio, field notes
Document analysis (schemes of work, teachers’ guides, learners’ textbooks)	Fieldnotes
Transcription and translation	Lesson transcripts, interview transcripts
Generating single-page summaries	Lesson graphs
Segmenting lessons into episodes	Episode summaries
Coding	Coded transcripts (MS Word, ATLAS.ti)
Querying	MS Word reports, Excel reports

3.6 Issues of trustworthiness and credibility

Yin (2016) highlights three major aspects of trustworthiness and credibility in qualitative research as transparency, methodic-ness and adherence to evidence.

3.6.1 Transparency

During this study, the procedures followed during data collection and analysis were documented as summarized in Table 3-15 and kept for further scrutiny. The recorded classroom data was also made available to the study participants. Going through the videos with the participants also helped to build rapport with them as noted from the comments made by the Standard 1 teacher during an interview, in Excerpt 3-3.

243. R: So, another question that I would like to ask now is about, aah, the first time that I started recording your lessons—when the camera came for the first time in the classroom—how did you feel? *[Laughs]*

244. T: *[Laughs]*

245. R: Because that time I don't know if your lessons were previously recorded.

246. T: No. Since I started teaching, I haven't been recorded. So, on that day, *[sighs]*, some little anxiety was inevitable, thinking: 'what's next?' Also, to me, I felt it's alright because now, I see: 'Oh, was this how the lesson progressed? There I was supposed to do this, and there I could have done it like this.' So, it helped me, because you came afterwards, but if you had just recorded the videos and disappeared, *[laughs]* I couldn't have known that 'here there was a problem, and here there was a problem.' But it wasn't a major issue for me.

Excerpt 3-3: Standard 1 teacher's response regarding the effect of the camera in the classroom.

By saying: "...if you had just recorded the videos and *disappear*..." in Utterance 246 of Excerpt 3-3, the teacher signified how she would have interpreted lack of a follow-up after collecting the classroom data. Though she did not express it explicitly, it seems she would have somehow felt uncertain about the purpose of the recorded videos if she had never seen them again.

Participant verification

The follow-up interviews were used for verifying some of the observations done in the classroom. Since the interviews were conducted after some preliminary analysis, the teachers were asked to confirm if the observations that were made represented what they usually do. To enhance the credibility of the findings, the period that had elapsed between the lesson

observation and the follow-up interviews was compensated by the use of video stimulated recall Nguyen et al (2013).

3.6.2 Methodic-ness

Methodic-ness was achieved by following an orderly routine at each stage of the data collection and analysis. The data analysis procedures were rigorously followed and documented as described in sections 3.5.4 and 3.5.7.

As highlighted by Schoenfeld (2007) the behaviour of study participants is highly affected by the context. As such, it is often necessary to observe a phenomenon using multiple lenses—that is, triangulation. In this study, data generated through classroom observations were substantiated with video simulated recall interviews and document analysis. Recording more than one lesson also helped to determine if there were notable changes in the teacher's actions from the first to the last lesson that would likely be attributed to the presence of the camera.

3.6.3 Adherence to evidence

The findings in this study were presented based on the evidence cited in the excerpts and accompanying figures. As discussed in section 3.5.7, the use of ATLAS.ti qualitative data analysis software enhanced the access to data, making it easier to notice patterns for one participant or any desired number of participants within the shortest time possible.

3.7 Ethical Considerations

In this study, ethical issues were handled by following procedures that were acceptable by the University of Malawi at the time of data collection. This was done by obtaining proper clearance before collecting data, followed by continued reflexivity throughout the data analysis process.

3.7.1 Permission

The process of seeking permission was top-down, beginning from the highest office in the district before any contacts were made with the study school (see Appendix 23 and Appendix 24). When carrying out research in public schools, Wanat (2008) emphasized on the need for observing the authority hierarchy. She indicated that gatekeepers at the top may not grant approval to carry out the study if the researcher started with seeking acceptance from the lower levels of the hierarchy.

3.7.2 Informed consent

Consent was also sought from the participants before data of any kind was collected from them. A key aspect that was observed was the cultural dimension of carrying out research in Africa as laid out by Wasunna, Tegli, and Ndebele (2014). Even though consent was supposed to be technically sought from the participants, in Africa, decisions are constructed socially and communally through negotiation. The communal approval and communal consent in African rural settings is culturally verbal (Tindana, Kass, & Akweongo, 2006). At the study school, this required the headteacher meeting with heads of the three sections of the school—the infant section (Standards 1 and 2), the junior section (Standards 3 and 4), and the senior section (Standards 5 to 8). The heads of sections were briefed about the objectives of the study and consented before meeting the concerned teachers. Similarly, after commencing data collection, all communication regarding the scheduling of visits was done through the headteacher, and not directly with the teachers. The last part of the process involved the headteacher informing the chairperson of the parents-teachers association who regularly visited the school to be aware of the research activity.

After the cultural endorsement from the gatekeepers, the teachers were met and informed about how the data will be collected and used. They were also informed that participation was

voluntary, and they had the right to withdraw from the study at any point. The explanation was presented to them on a written form, which they signed and kept a copy (see Appendix 22).

3.7.3 Identity of participants

Since the interest for this study was on the teacher, the video camera was positioned at the back of the classroom to chiefly focus on the teacher's actions during data capture. Identities of individuals shown in pictures were made anonymous by shading their faces. All names of learners appearing in transcripts are pseudonyms. The school's identity was also concealed by a pseudonym while the teachers were referred to using their classes. Despite a rigorous description of the selection process, the school belonged among a group of top-performing schools in the district, that are geographically spread, hence could not be easily identified.

3.7.4 Reflexivity

Rather than taking research ethics as a one-off activity achieved through letters of permission and consent, this study took the issue of ethics from a reflexivity standpoint. I introduced myself to the teachers as a PhD student who would like to learn from them. Despite explaining my role, it seemed the teachers could not quickly understand my research role for two reasons.

First, the teachers seemed to have wondered about what someone from the university might learn from a primary school. So, despite explaining to them the objectives of the study, they seemed to be curious to see how the learning process unfolds from a research perspective. Secondly, the teachers seemed to have been used to classroom observations that were supervisory in nature. Knowing this, whenever an opportunity arose, I had to remind the participants regarding my role as a researcher and not a supervisor.

The two challenges were clarified through reflective journaling of interesting or surprising observations made during the lessons that was quickly followed by unstructured interviews with the teachers to learn more from them regarding the observations. It was through these

informal interviews that were conducted at the earliest opportunity with the teachers that they grasped my role as a researcher. The participants could notice from the informal interviews that my focus when observing the lessons was different from what they would hear from an inspector who had supervised their lessons.

3.8 Piloting

The pilot study was done with a Standard 2 teacher at a purposively sampled primary school. Just like the main study, the school for the pilot study was chosen based on the performance of learners during PSLCE. One teacher was observed during the week when she was teaching the addition of numbers. The findings from the pilot study helped in modifying the research instruments, including the best positioning of the camera when recording lessons. The findings also gave some insights on the volume of data generated with the MPM analytical framework. The observations made from the analysis of the Standard 2 teacher in the main study confirmed that the volume of data noted during the pilot was not unique to the mediational profile of the observed teacher. This made it necessary to plan for switching to the use of qualitative data analysis software. The teacher who was involved in the pilot study also pointed out the influence of school inspectors on her usage of mediational means—a fact that was not mentioned by the other participating teachers.

3.9 Chapter summary

This chapter presented the theoretical assumptions that influenced the design of the study. Meaning making was guided by sociocultural theory. This theory helped to understand the role of the teacher as well as the nature of mathematics as a scientific discipline. The qualitative case study design helped to understand the contextual issues surrounding the mediation mathematics in the early years of primary school classes. Data collection and analysis was

systematically done to ensure the trustworthiness and credibility of the findings presented in the next chapter.

CHAPTER 4

FINDINGS

4.1 Introduction

This chapter presents the findings from the four study participants based on the analysis of transcripts of recorded lessons and interviews with the teachers, field notes, the mathematics teachers' guides, as well as the learners' mathematics textbooks for Standards 1 to 4. As stated in Chapter 1, the study was guided by the main research question: *How do teachers mediate mathematics during the early years of primary school in Malawi?* During the study, mediation of mathematics was interpreted as how teachers used artefacts, inscriptions, as well as talk and gesture when working with tasks and examples in the classroom (Venkat & Askew, 2018). As such, the main question was answered by considering the following three subsidiary research questions (RQs):

RQ1: How do teachers in the early years of primary school select tasks and examples during mathematics lessons?

RQ2: How do teachers use artefacts, inscriptions, and explanations to represent mathematical concepts and processes in the early years of primary school?

RQ3: What is the rationale behind the teachers' choice of tasks and examples, artefacts, inscriptions, and explanations used during lessons?

The findings for the above three research questions across cases have been presented in section 4.2, followed by the findings for each of the four teachers in sections 4.3, 4.4, 4.5, and 4.6, in

the order of class levels. For each section, say 4.3, the first subsection (4.3.2) focuses on teachers' selection of tasks and examples (RQ1) as well as the rationale behind the choices made (RQ3). The rest of the subsections, say 4.3.3, 4.3.4, and 4.3.4, focus on teachers' use of mediating artefacts, inscriptions, talk and gesture respectively (RQ2), as well as the rationales for their choices of each means of mediation (RQ3).

While the first subsection 4.3.2 presents the observed tasks and examples, the subsection 4.3.4 on teacher's mediating talk and gesture, gives more details on the methods that the teachers used when generating solutions and the possible connections that were made noticeable to the learners. The observations presented in this chapter have been discussed with respect to literature in Chapter 5.

4.2 Comparison across cases

This section is aimed at giving a synopsis of how the observed teachers worked with various mediational means. As stated in the methodology chapter, all the lessons were observed during the first term of the school year. Since teachers in Malawi are often assigned to teach a particular grade level, each teacher was working with the set of learners in the classroom for the first time, except for learners who were repeating the class level after failing examinations at the end of the previous school year.

4.2.1 Mediating tasks and examples

This subsection describes the tasks and examples observed during the lessons, followed by the rationale behind the choices made by the teachers. The discussion in this section focuses on the types of tasks, as well as the sequencing and duration of the tasks across the lessons.

Nature of tasks and examples observed across cases

Apart from tasks involving the addition of numbers, counting tasks were also observed during the introduction of some lessons in Standards 1, 2 and 3. Some counting in Standards 1 and 2 was done through singing number songs.

Types of tasks

The observed tasks mainly differed in the roles of the teacher and the learners when executing them. Table 4-1 categorises the tasks according to the way the teacher and the learners worked to execute them.

Table 4-1: Types of tasks used by the four teachers

Type of task	Description
Teacher and whole class	These tasks were completed through teacher-led discussions where all learners in the class were given opportunities to contribute to various parts of the major task until the solution was found.
Learner in front	A learner was chosen to work out the entire problem on the chalkboard. The teacher's intention was for the learner to assume the role of the teacher and lead the discussion with the whole class.
Group work	These were tasks that were completed with learners contributing ideas within their groups leading to the final solution. Learners seemed to know their pre-assigned groups.
Individual work	Individual work was completed in the learners' notebooks and marked by the teacher.
Pair work	Learners worked in pairs to find solutions to given problems.

Some tasks involved a blend of several classroom interaction strategies. For example, during the last episode of Standard 3 Lesson 1, the teacher started by working with the class in finding the solution to one problem and asked two learners to take turns solving the last two problems. Some classwork was done in groups for the sake of sharing resources, but not necessarily to share ideas. This was noted during Lesson 2 of Standard 3 where the learners were seen sharing abaci in groups when doing the first task with the teacher. After finishing the first task, the teacher asked the learners to go into their groups to do the second task.

Nature of examples

Except for Standard 4, the teachers worked with pairs of addends that were described by the maximum sum in the curriculum. The nature of examples that the teachers worked with across the four classes have been summarised in Table 4-2.

Table 4-2: Nature of examples across the lessons

Class	Number of addends	Maximum sum	Written presentation of problems
Standard 1	2	5	Horizontal ($a + b$) and vertical $\left(\begin{array}{r} a \\ + b \\ \hline \end{array} \right)$ layout
Standard 2	2	50	Vertical layout under headings T and O
Standard 3	2	600	Vertical layout under headings H, T and O
Standard 4	4	9,999	Vertical layout under headings Th, H, T and O

Table 4-2 shows that the number of digits for each addend was limited to a single digit in Standard 1, two digits in Standard 2, three digits in Standard 3, and four digits in Standard 4. All the examples solved in Standards 1 and 2 had to be formulated in such a way that the process of finding the solution would not require regrouping (Malawi Institute of Education, 2012c). Problems requiring regrouping were done in Standards 3 and 4. The problems in the last lesson of Standard 4 were all presented as word problems.

Sequencing of tasks

Despite the slight variations observed in Standard 4, the four teachers generally followed the same order of lesson tasks from the introduction to the closure of the lesson. During interviews, they all agreed on the observed pattern. Due to this similarity, lessons for some of the classes ended up being segmented into the same number of episodes during analysis. Table 4-3 shows the sequencing of tasks in a typical lesson using the case of Standard 1.

Table 4-3: The sequencing of tasks in a typical lesson

Episode	1	2	3	4
Task	Review of previous learning	Solving one or more examples	Solving an example	Solving two or three examples, and homework
Nature of task	Whole class	Whole class	Group work	Individual work

As shown in Table 4-3 the introduction was followed by an example solved by the teacher and the whole class. This was generally followed by one or more examples done in groups. In some cases, all the groups worked on one example or each group worked on a unique example. The solutions found through group work were verified on the chalkboard by some chosen learners or by the teacher and the whole class. Group work was mostly followed by individual work written in learners' notebooks and marked by the teacher. The solutions were also verified on the chalkboard by some learners or by the teacher and the class. In some cases, the teacher ended the class by giving homework. Except for the Standard 1 teacher, unmarked notebooks for individual work were collected to be marked at another time.

Even though the lessons flowed in a similar sequence, the Standard 3 lessons were the most consistent in structure and sequencing (see Figure 4-4), followed by Standard 2. The Standard 4 lessons had the largest variation in structure and sequencing, followed by Standard 1. Only the Standard 4 and Standard 1 teachers included some episodes with addition problems that were solved mentally by the learners.

Duration of tasks

Although there were differences on the amount of time that the teachers used for different types of tasks across the lessons, the average timing, however, shows that the teachers for Standards 3 and 4 used fairly similar amounts of time for all types of tasks compared to the teachers for Standards 1 and 2 (see Figure 4-1). In Standards 1 and 2, much of the lesson time was spent on tasks done by the teacher and the whole class.



Figure 4-1: Average time spent on various types of tasks in each class and across the four classes (Source: Researcher).

Regarding the overall teaching time, the lessons by all the teachers were generally longer than the official time allocated to a single mathematics lesson. For each school day, the official timetables for all the classes indicated two mathematics lessons with a duration of 30 minutes each for Standards 1-2 and 35 minutes each for Standards 3-4. Since the teachers taught mathematics once each day, they possibly combined the two separated periods for each day. In that case, the combined periods were supposed to take 60 minutes (1 hour) for Standards 1 and 2, and 70 minutes (1 hour 10 minutes) for Standards 3 and 4.

The rationale for the teachers' selection of tasks and examples

The teachers generally agreed that they followed the suggestions from the teachers' guide. In the teachers' guide, the mathematics content for each class was divided into units, and it provided the content and guidelines to be followed when carrying out the activities under a particular unit (see Figure 4-47 and Figure 4-66). The teachers' guide indicated the number of lessons required to complete the tasks within each activity. The teachers' guide also provided references to the corresponding tasks in the learners' textbooks where tasks to be given as

individual work could be found. The Standard 4 teacher moved together with the learners from their copies of mathematics textbooks when presenting examples to work with. This made it possible for the Standard 4 teacher to give work to the learners by just referring them to the required page. The Standard 1 teacher, on the other hand, indicated that she formulated the examples to use by herself, as long as the sum was within the required limit specified in the curriculum (see Excerpt 4-3). The teacher felt that some of the illustrations showing quantities of items in the Standard 1 textbook were confusing for the learners. The Standard 2 and Standard 3 teachers copied the problems from the textbook and presented them on the chalkboard.

Regarding the number of tasks and examples, the major factor for the teachers' decision was class size. Except for the Standard 3 teacher, group work was used to do as many examples as possible, while two or three examples were done as individual work marked by the teacher.

4.2.2 Mediating artefacts

Nature of artefacts

The nature of artefacts used by the teachers across the four classes have been summarised in Table 4-4.

Table 4-4: Nature of artefacts used across the class levels

Class	Artefacts used
Standard 1	Framed counters, prewritten papers, books, stones, leaves, sticks
Standard 2	Framed counters, prewritten papers, place-value boxes
Standard 3	Framed counters, spike abaci
Standard 4	Framed counters, prewritten papers, loose counters

It can be seen in Table 4-4 that each of the four teachers worked with framed counters during their lessons. The counters were mostly made from pieces of grass straw or bottle-tops with a

string running through them and fastened to two ends of a frame. See example in Figure 4-2, which shows a teacher carrying counters fitted on a bow-shaped frame.

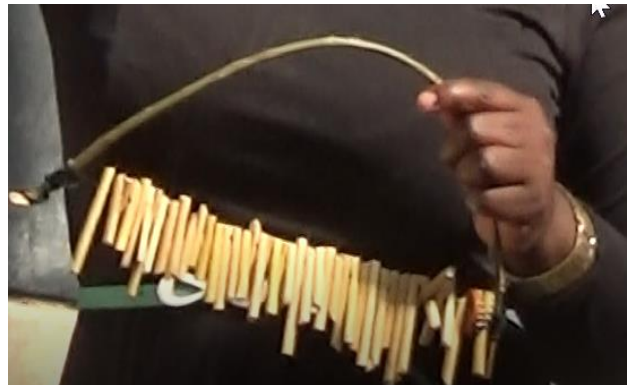


Figure 4-2: Example of framed counters (Source: Researcher).

Learners were seen with different quantities of counters made from the materials they would easily find, as shown in Figure 4-3.

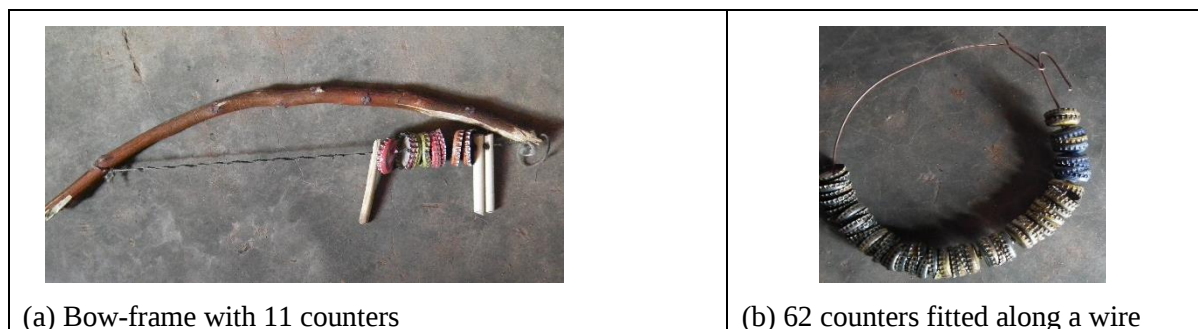


Figure 4-3: Variety of framed counters used by learners (Source: Researcher).

When introducing the framed counters in Standard 1, the teacher made a sample and asked the learners to make theirs with the help of their parents. The bow shape of the frame is so common, such that Saka and Roberts (2018) called the artefact “the Malawian bow-abacus”. In addition to the framed counters, the Standard 1 teacher also used other physical objects such as books, leaves, stones and sticks during her first lessons before switching to exclusive use of framed counters during her last two lessons. The Standard 2 teacher also worked with place-value

boxes when introducing place-value addition. The Standard 3 teacher used spike abaci when introducing addition involving regrouping.

Teachers' use of artefacts

When finding the sum, the teachers generally used the count-all strategy for addition. When working out solutions with the whole class, the teachers ensured that all the learners were participating in the counting with their framed counters. For learners who had not brought their counters, the teachers asked them to use their fingers.

The teachers mostly worked with the artefacts in parallel with their corresponding inscriptions as well as talk and gesture. The affordances and constraints associated with the teachers' use of artefacts have been discussed in section 5.3 of the next chapter.

The rationale for the teachers' use of artefacts

The teachers' choice of artefacts was mostly driven by their availability. As stated by the Standard 1 teacher, single-use counters from the local environment (such as sticks, leaves, stones) required some time to collect them. To save this time, the teacher encouraged the learners to make their own framed counters. The teachers also worked with other types of artefacts, such as place-value boxes and spike abaci, based on the suggestions from the teachers' guide.

4.2.3 Mediating inscriptions

Nature of inscriptions

The inscriptions used by the four teachers were mostly structured mathematical statements written on the chalkboard. It was only during the first three Standard 1 lessons where the inscriptions also included drawings of objects such as leaves, stones, and sticks (see Figure 4-18).

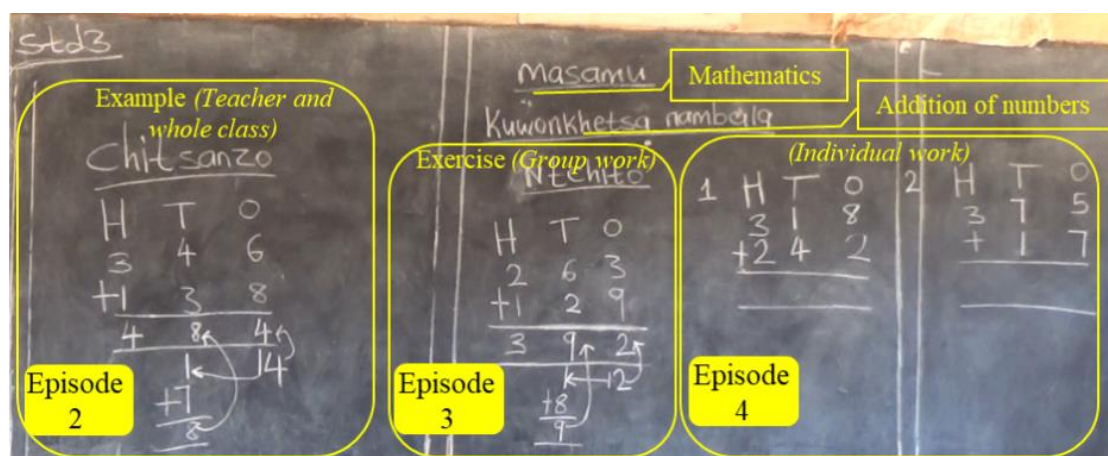


Figure 4-4: Inscriptions of tasks and examples done during Lesson 3 of Standard 3 (Source: Researcher).

Teachers' use of inscriptions

All the teachers mostly used the chalkboard for presenting tasks and examples. On the chalkboard, the work done by the teacher and the whole class was indicated as *chitsanzo* [example] while the group work or individual work was indicated as *ntchito* [exercise] as shown in Figure 4-4. Except for Standard 2, the teachers also used inscriptions for showing the method followed to obtain the solution. As shown in Figure 4-4, the method was shown using arrows as well as inscriptions of calculations written below the bottom horizontal bar of the addition problem.

Learners were mostly asked to participate by taking turns reading inscriptions presented on the chalkboard. When generating or verifying solutions of problems on the chalkboard, learners took turns in writing the inscriptions leading to the answer. It was during individual work when the learners were asked to write in their notebooks. The teachers for Standards 1 and 2 reminded learners to write the routine details, such as the subject, topic, and date. Some learners in these classes were observed taking time just to write this information, which was followed by copying the given problems before they could start working out the solutions.

The rationale for the teachers' use of inscriptions

The curriculum was designed in such a way that learners were first introduced to a set of numbers followed by arithmetic operations, such as addition, on those numbers. For instance, the first two lessons of Standard 2 were observed during the sixth week of the first term. By this time, the learners had just been introduced to the numbers 10 to 20, as specified in the curriculum. Thus, it can be assumed that the learners were still in the process of getting familiarised with numbers greater than 10, and at the same time, they were supposed to add these numbers. It was probably due to this reason that after presenting the example on the chalkboard, the rest of the inscriptions leading to the expected answer were mostly done by learners. Thus, the teachers used chalkboard inscriptions as an opportunity to strengthen the learners' mastery of writing the new numbers that they had just learnt. As regards the preliminary details (such as topic and date) that the learners were expected to write down in their notebooks, the Standard 1 teacher explained to the learners that these would enable their parents to know that they were learning addition at this stage.

4.2.4 Mediating talk and gesture

Some similarities and differences were noted on the teachers' mediating talk and gesture for providing methods for generating solutions, building mathematical connections, and advancing learning connections.

Teachers' mediating talk and gesture for providing methods for generating solutions

The teachers' talk mainly comprised short statements that often ended as a question. The statements often ended by asking "alright?" The teachers also often repeated statements, especially in Standard 1, where getting the attention of the learners required the teacher's additional effort.

All the teachers used the count-all strategy of addition and used unit counting when working with each addend during the calculations. In some instances, learners offered the required solution quickly, but the teachers restrained such ones (see Excerpt 4-19). Instead, the teacher and the class went through several steps to arrive at an answer that some of the learners or the entire class had already mentioned earlier but was ignored (see Excerpt 4-20).

Teachers' mediating talk and gesture for building mathematical connections

During the lessons, more connections within examples were noted compared to connections across examples.

Use of multiple means of mediation

The teachers made strong connections within examples by mostly using multiple means of mediation for the same example. The teachers mostly accompanied their talk with a pointer to what was written on the chalkboard, thereby connecting their talk and gesture with inscriptions. For instance, in Figure 4-5, the Standard 2 teacher moved her hand back and forth between the two bundles in the place-value box and the inscription of 2 under tens (T). In the figure, the teacher's placement of the place value boxes beneath the numbers being mediated enhanced visualisation of the mathematical connections.

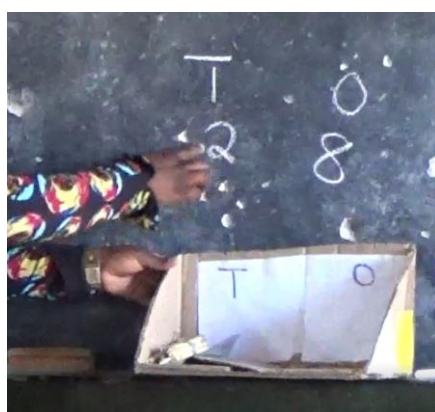


Figure 4-5: Connecting artefacts and inscriptions with talk and gesture (Source: Researcher).

Even though the teachers made several connections within examples, there were some missed opportunities for making connections across examples. Only the Standard 2 teacher made connections across examples explicit during her first two lessons as shown in Figure 4-58 and her accompanying talk in Excerpt 4-22.

Use of language

Some notable observations were also made in the teachers' use of the language of teaching and learning during the early years of primary school in Malawi, Chichewa. The language-related issues stemmed from the translation of mathematical terms from English to Chichewa in the curriculum, as discussed in Chapters 2 and 5. For example, the teachers made references to the place-value headings Th, H, T, and O for thousands, hundreds, tens, and ones, with their corresponding Chichewa equivalents of *masauzande*, *mahandirede*, *mateni*, and *mawani*. The learners managed to associate Th, H, T, and O with the Chichewa equivalents even though they would not readily make conceptual connections between the visual and verbal representations of the place-value notations. Since the teachers' guide is in English, it only gave instructions such as: "Discuss the meaning of T and O" (Malawi Institute of Education, 2012c, p. 10), leaving the language technicalities to the teacher.

Despite the technical challenges, the teachers shared the same language when referring to similar concepts and processes. For instance, all the teachers worked with problems involving the addition of zero in the same way, and they all referred to zero as "palibe" [nothing]. The teachers in the upper classes had assumptions on what was done in lower classes regarding the addition of zero, as said by the Standard 3 teacher during an interview (see Excerpt 4-1).

155. T: They have had that explanation from Standard 1. Because in Standard 1, when the teacher asks them: "Let us count zero!" The children were saying: "zero!" And when the teacher would ask: "What is zero?" The children would go like: "Nothing!" So, they would know that "if there's nothing then there is zero".
--

Excerpt 4-1: Teacher's explanation of zero from Standard 1.

Teachers' mediating talk and gesture for advancing learning connections.

Regarding the evaluation of learners' offers, the teachers mainly asked the class to judge the correctness of the offers from their classmates. After the correct solution was given by one of the learners, the teachers offered them appropriate positive reinforcement using a variety of hand-clapping styles.

Among the four teachers, the Standard 1 teacher explored learners' ideas further and allowed them to make many attempts. The Standard 4 teacher achieved strong learning connections by capitalising on learners' common errors. Instead of waiting for learners to make mistakes and correct them at that point, she deliberately made the errors on the chalkboard, which prompted the class to react against the teachers' error. In some cases, the Standard 4 teachers' deliberate errors sounded so sincere, to such an extent that the learners took a few seconds to realise that the teacher was deliberately luring them to an incorrect answer. This approach allowed learners to take note of the common errors and misconceptions associated with the place-value addition algorithm.

4.2.5 Summary of the usage of mediational means across cases

The teachers' lessons shared many similarities in the way tasks and examples were presented and sequenced. Instead of teaching two short mathematics lessons as provided on the timetable, the teachers mostly combined the two lessons into one extended lesson. The four teachers also shared several similarities in their usage of artefacts during the lessons, mostly working with framed counters. They used chalkboard inscriptions presenting tasks as well as for recording the method for generating solutions. When working out the required solutions, all the teachers used the counting-all strategy for addition.

The next section (4.3) presents the usage of mediational means by the Standard 1 teacher.

4.3 Use of mediational means in Standard 1

This section presents how the Standard 1 teacher worked with different mediational means across the six lessons. As stated in the methodology chapter, all the lessons were observed during the first term of the school year. Five of the six lessons were observed during the 10th week while the sixth lesson was observed during the 11th week of the term. Each of the six lessons had four episodes that were segmented based on the tasks that teacher and the class focused on.

4.3.1 An overview of Standard 1 lessons

During the lessons, the Standard 1 teacher was introducing the concept of addition for the first time after the learners had been dealing with counting and writing numbers up to 5 during the preceding weeks. By the end of the school year, the Standard 1 learners were expected to work with addition of numbers in the range 0 to 9.

Lesson 1

During the first episode of Lesson 1, the teacher discussed the meaning of the concept of addition with the class. This was followed by teaching the learners how to write the plus and equal signs. The second episode focused on finding the sum of two books and one book in groups followed by a whole class discussion on how to write “two plus 1 equals 3” on the chalkboard. During Episode 3, the teacher asked learners to add four stones and one stone in their groups and asked them to try out writing “four plus 1 equals 5” on the chalkboard. The teacher illustrated $4 + 1 = 5$ by drawing circles representing stones on the chalkboard. The last episode also started with a group task of finding the sum of two leaves and two leaves followed by drawing the leaves on the chalkboard and writing their corresponding numbers below them. The progression of Lesson 1 has been visually presented in a single-page lesson graph in Appendix 1.

Lesson 2

The first episode focused on a review of the plus and equal signs learnt during Lesson 1. In the second episode, the teacher asked learners to find the sum of two sticks and three sticks. The teacher drew the sticks on the chalkboard and asked the learners to write the corresponding addition statement “two plus three equals five” on the chalkboard. In Episode 3, learners were asked to work out $2 + 1$, $1 + 1$, and $3 + 0$ mentally. The last episode was individual work with two problems ($2 + 1$ and $3 + 1$) that were presented using drawings. The learners were also given one problem ($1 + 1$) using drawings as homework. The lesson graph in Appendix 2 depicts how the activities in Lesson 2 were done.

Lesson 3

This lesson also started with a review of the plus and equal signs in Episode 1. In Episode 2 the teacher asked the class to present “2 leaves plus 1 leaf equals 3 leaves” on the chalkboard using drawings and a written statement below the drawings. During Episode 3, the teacher presented a chart with drawings of three balls and two balls and asked learners to present the sum on the chart. In the last episode, learners were given drawings on a chart to be worked out as in the previous episodes. The tasks done during Lesson 3 have been visually presented in the lesson graph in Appendix 3.

Lesson 4

Episode 1 was a review of previous learning during which learners were asked to write a plus sign on the chalkboard. The class was then asked to present “2 balls plus 2 balls equals” on the chalkboard. This was followed by a discussion of the strategies used to present the answer. During Episode 2, the teacher worked out the solution for $2 + 0$ with the whole class. The third episode was groupwork during which learners were given papers with $2 + 1$, $3 + 1$, $4 + 1$, $1 + 1$, $5 + 0$, $1 + 2$, $3 + 0$, $2 + 2$, $2 + 3$, $4 + 0$. The solutions found by the groups were verified by the whole class using counters. In Episode 4, the learners were asked to work out $3 + 2$, $0 + 5$

and $2 + 1$ in their notebooks and were marked by the teacher. Appendix 4 shows the lesson graph for Lesson 4.

Lesson 5

The teacher started Lesson 5 by asking the learners to write an addition statement on the chalkboard. At the end of the first episode, the class had come up with " $2 + 2 = 4$ " on the chalkboard. In Episode 2, the teacher posted a paper with " $1 + 0 =$ " on the chalkboard and asked the learners to write the answer. The correct answer that was written by one learner was then verified by the whole class using counters. During Episode 3, groups were given papers in with $2 + 2$, $4 + 0$, $1 + 1$, $3 + 2$, $1 + 3$, $0 + 4$, $2 + 1$, $2 + 0$. The solutions found by the groups were verified by the teacher and the whole class using counters. During the last episode, learners were asked to workout $3 + 1$, $4 + 1$, and $0 + 3$ in their notebooks and were marked by the teacher. The teacher and the class verified the solutions for the problems using counters. The activities done during Lesson 5 have been shown in Appendix 5.

Lesson 6

The first episode focused on review of plus and equal signs. In Episode 2, the teacher introduced the vertical notation of addition (place-value layout) using the example "0 plus 1 equals" and discussed the correct alignment of addends and the answer. The teacher pasted a chart paper with a vertical representation of $1 + 3$ and discussed the presentation of the answer. Episode 3 involved groupwork during which learners solved $1+1$, $4+0$, $2+2$, $3+1$, $5+0$, $3+2$, $1+2$, $1+3$, $0+1$, $0+2$, and $2+1$. The solutions found by the groups were later verified by the whole class. In Episode 4, learners were asked to work out $1 + 4$, $2 + 0$, and $3 + 1$ presented vertically on the chalkboard. The lesson graph for Lesson 6 has been presented in Appendix 6.

4.3.2 Mediating tasks and examples

Nature of tasks and examples observed in Standard 1

The tasks in all the lessons involved adding two numbers with a sum not exceeding 5. The major difference across the lessons was on how the tasks were presented.

Types of tasks

The teacher's presentation of tasks across the six lessons has been summarised in Table 4-5 that follows.

Table 4-5: The presentation of tasks across the six Standard 1 lessons

Lesson	1	2	3	4	5	6
Presentation of tasks	Verbal, physical artefacts	Verbal, physical artefacts, drawings	Drawings and written statements	Written horizontal addition statements	Written horizontal addition statements	Written vertical addition statements

During the first lesson, the tasks were presented entirely using physical artefacts (books, stones, and leaves). During Lesson 2, the learners performed addition using sticks in their groups, but the individual task was given using drawings (see Figure 4-11). The use of written statements for presenting tasks was done from the third lesson up to the last lesson.

Duration of tasks

The durations of various types of tasks in Standard 1 have been presented in Figure 4-6. Starting from the fourth lesson of Standard 1, the tasks that were done as group work were followed by whole-class verification of the solutions, as shown under the section marked 3.2 in Table 4-6 to Table 4-8. The whole-class verification for Lessons 4 to 6 took an average of 22 minutes of the lesson time. This ultimately resulted in the lessons being much longer than the officially set duration of 30 minutes for Standards 1-2.

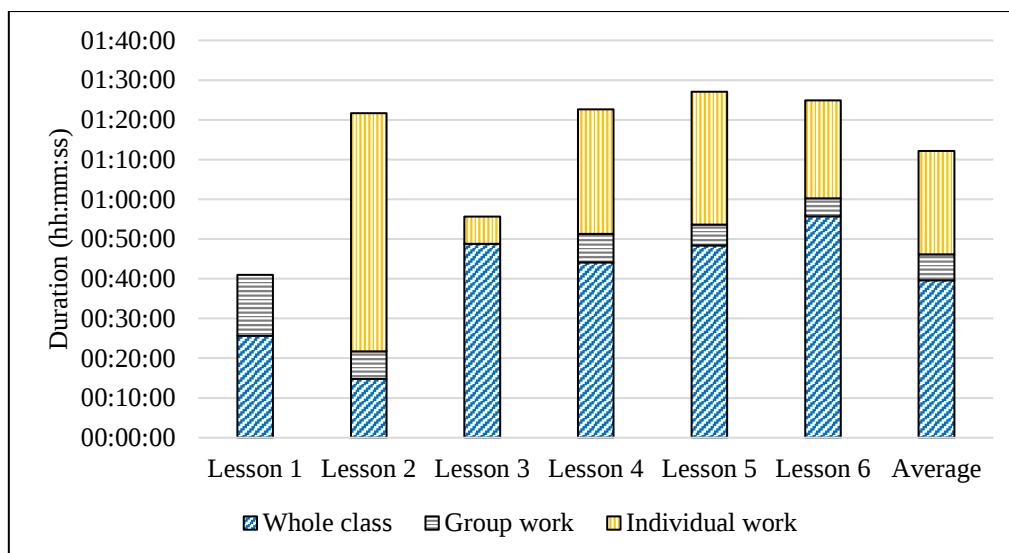


Figure 4-6: The time spent on various types of tasks across the six Standard 1 lessons (Source: Researcher).

Lesson 2 was a continuation of the first lesson that was conducted earlier on the same day. However, despite being a continuation of the first lesson, Lesson 2 took 1 hour and 22 minutes. Much of this time (1 hour) was spent on marking individual work. On average, the class time was dominantly shared between whole-class tasks and individual work (see Figure 4-7).

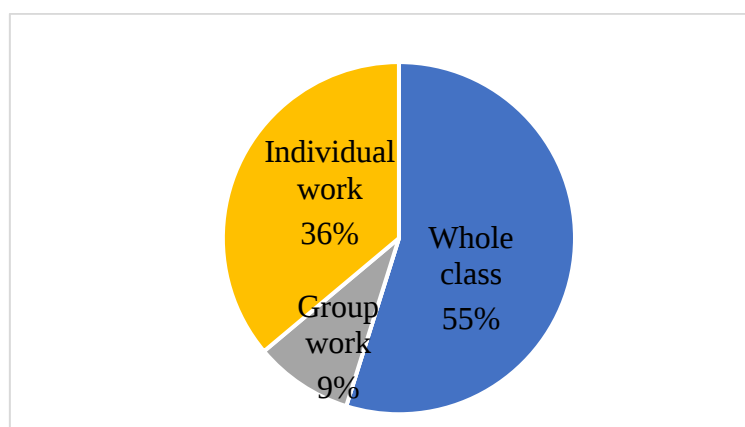


Figure 4-7: Average duration of task types in Standard 1 (Source: Researcher).

Lesson sequence

Each of the six lessons flowed in the same sequence, each with 4 episodes that were structured as shown in Table 4-3 under section 4.2.1. However, there was a slight variation in the sequencing of tasks during the first three lessons when most of the concepts were being newly introduced. The last three lessons were consistently structured. After the introduction, the lessons proceeded with a whole-class example, followed by groupwork and individual work, as presented in Table 4-3.

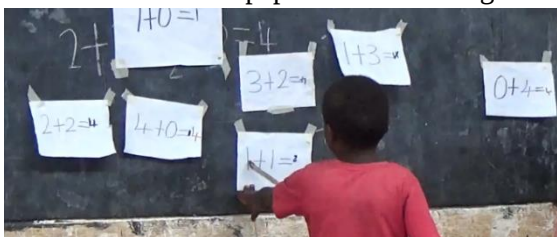
Except for Lesson 1, the lessons were introduced with a review of previous learning in Episode 1. The introduction took less than 5 minutes for the first three lessons, but took more than 6 minutes for the last three lessons; getting close to 10 minutes for Lessons 4 and 5. This was mostly followed by one or two examples done by the teacher and the whole class. Apart from the first three lessons, the third episode involved learners finding solutions to a unique problem in their groups. After finding the solutions, each group was asked to choose a representative to present their work to the class. The similarity of the activities in Episode 3 across the last three lessons has been illustrated in the excerpts of their respective episode summaries shown from Table 4-6 to Table 4-8.

Table 4-6: Summary of Episode 3 of Lesson 4

3	3.1	Finding $2 + 1$, $3 + 1$, $4 + 1$, $1 + 1$, $5 + 0$, $1 + 2$, $3 + 0$, $2 + 2$, $2 + 3$, $4 + 0$ (Group work)	Distributes papers with prewritten addition statements ($2 + 1$, $3 + 1$, $4 + 1$, $1 + 1$, $5 + 0$, $1 + 2$, $3 + 0$, $2 + 2$, $2 + 3$, $4 + 0$) to groups. Asks learners to use the leaves placed in their groups to work out the answer together and write it down. 
	3.2	Verifying solutions for $2 + 1$, $3 + 1$, $4 + 1$, $1 + 1$, $5 + 0$, $1 + 2$ and $3 + 0$ (Teacher and whole class)	Asks group representatives to stick the papers with their given statements on the chalkboard. Asks a learner to read out the written papers pasted by all the groups, one at a time.  The teacher works with the class to verify the solutions given by the groups using counters. The class verifies $2 + 1$, $3 + 1$, $4 + 1$, $1 + 1$, $5 + 0$, $1 + 2$ and $3 + 0$

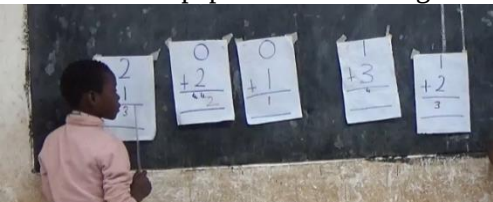
As shown in Table 4-6, Episode 3 of Lesson 4 started with learners working independently in groups to find solutions to the problems given to them on a piece of paper. The teacher asked a representative from each group to stick their paper on the chalkboard. Afterwards, the teacher asked one learner from the class to read all the pasted papers.

Table 4-7: Summary of Episode 3 of Lesson 5

3	3.1	Finding $2 + 2$, $4 + 0$, $1 + 1$, $3 + 2$, $1 + 3$, $0 + 4$, $2 + 1$, $2 + 0$ (Groupwork)	Distributes papers with prewritten addition statements ($2 + 2$, $4 + 0$, $1 + 1$, $3 + 2$, $1 + 3$, $0 + 4$, $2 + 1$, $2 + 0$) to groups. Asks learners to work together to find the answer to their given problem and write it down. 
	3.2	Verifying solutions for $2 + 2$, $4 + 0$, $1 + 1$, $3 + 2$, $1 + 3$, $0 + 4$, $2 + 1$, $2 + 0$ (Teacher and whole class)	Asks group representatives to stick the papers with their given statements on the chalkboard. Asks group representatives to remain in front and wait for their turn to present their solution to the class.  The teacher works with the class to verify the solutions given by the groups using counters.

During Episode 3 of Lesson 5 and Lesson 6—shown in Table 4-7 and Table 4-8 respectively—the teacher asked the group representatives to read their papers they had just pasted on the chalkboard, rather than asking one learner to read all papers as it was done in Lesson 4.

Table 4-8: Summary of Episode 3 of Lesson 6

3	3.1	Finding $1 + 1$, $4 + 0$, $2 + 2$, $3 + 1$, $5 + 0$, $3 + 2$, $1 + 2$, $1 + 3$, $0 + 1$, $0 + 2$, and $2 + 1$. (Groupwork)	Distributes papers with vertical addition statements to groups. Asks learners to work together to find the answer to their given problem and write it down. 
	3.2	Verifying solutions for $1 + 1$, $4 + 0$, $2 + 2$, $3 + 1$, $5 + 0$, $3 + 2$, $1 + 2$, $1 + 3$, $0 + 1$, $0 + 2$, and $2 + 1$ (Teacher and whole class)	Asks group representatives to stick the papers with their given statements on the chalkboard. Asks group representatives to remain in front and wait for their turn to present their solution to the class.  The teacher works with the class to verify the solutions given by the groups using counters.

During an interview, the teacher outlined the way she sequenced the tasks during a typical lesson. The order can be noted from the underlined utterances in Excerpt 4-2.

- | |
|--|
| <p>140. T: Okay. At first, we have to model, that is, an <u>example</u>. We do an example with the learners, together with the learners. After that, it's when we now come to give the learners <u>work in groups</u>, so that you should know that 'have they mastered what I told them'? After giving them the work in their groups, it's when we give them the <u>work to do on their own</u> in exercise books now.</p> <p>141. R: The third step: As individuals. From there?</p> <p>142. T: So, after individual work, it's when you come to...aah...after <u>marking</u> it's when you come to <u>corrections</u>.</p> <p>143. R: Okay, then do corrections.</p> <p>144. T: That is how it flows.</p> <p>145. R: After that?</p> <p>146. T: After corrections, then we give them <u>homework</u>.</p> |
|--|

Excerpt 4-2: Sequence of task types in a typical Standard 1 lesson.

The sequencing of observed types of tasks shown in Table 4-3 agrees with the outline given by the teacher in Excerpt 4-2. Except for Lesson 1, the last episode was always individual work marked by the teacher. During some of the lessons, the teacher also gave homework. The routine was familiar to children such that one learner was observed taking out a notebook from her bag during the last quarter of Lesson 1, even though the teacher had not instructed the class to write.

Examples used in Standard 1

All the examples used during the six lessons had two addends with a sum not exceeding 5 as stipulated in the syllabus for the first term of Standard 1. The list of examples used across the six lessons has been presented in Table 4-9.

Table 4-9: List of examples used in Standard 1

	Lesson 1	Lesson 2	Lesson 3	Lesson 4	Lesson 5	Lesson 6
Episode 1	$+, =$	$+, =$	$+, =$	$+, 2 + 2$	$2 + 2$	$+, =$
Episode 2	$2 + 1$	$2 + 3$	$2 + 1$	$2 + 0$	$1 + 0$	$0 + 1, 1 + 3$
Episode 3	$4 + 1$	$2 + 1, 1 + 1,$ and $3 + 0$	$3 + 2$	$2 + 1, 3 + 1, 4 + 1, 1 + 1, 5 + 0, 1 + 2, 3 + 0, 2 + 2, 2 + 3, 4 + 0$	$2 + 2, 4 + 0, 1 + 1, 3 + 2, 1 + 3, 0 + 4, 2 + 1, 2 + 0$	$1 + 1, 4 + 0, 2 + 2, 3 + 1, 5 + 0, 3 + 2, 1 + 2, 1 + 3, 0 + 1, 0 + 2, 2 + 1$
Episode 4	$2 + 2$	$2 + 1, 3 + 1,$ and $1 + 1$	$2 + 2$	$3 + 2, 0 + 5, 2 + 1$	$3 + 1, 4 + 1, 0 + 3$	$1 + 4, 2 + 0,$ and $3 + 1$

Lesson 1 started with learning the writing of the $+$ and $=$ sign, asking the whole class to practice writing in the air. The rest of the examples were presented as quantities of objects to be added. For example, Episode 2 involved adding 2 books and 1 book, followed by writing $2 + 1 = 3$ on the chalkboard. Episodes 3 and 4 involved adding stones and leaves respectively.

There were slight changes in the way the examples were presented from Lesson 2 onwards. For instance, the examples in Episode 3 of Lesson 2 were presented as mental addition, asking the learners to quickly state the answer without using physical artefacts to find the sum. The examples in Episode 4 of Lesson 2 were presented as drawings of trees and stick diagrams of people with their corresponding numbers below them. The teacher started presenting the examples as structured mathematical statements from Lesson 4 onwards. The teacher introduced vertical addition in Lesson 6.

The rationale for the Standard 1 teacher's choice of tasks and examples

There are some reasons behind the Standard 1 teacher's choice and sequencing of tasks and examples.

The rationale for the selection of tasks

The teacher seemed to follow the suggestions in the Teachers' Guide for Standard 1 in her selection of tasks. For example, the first three lessons followed the instructions from the teachers' guide shown in Figure 4-8.

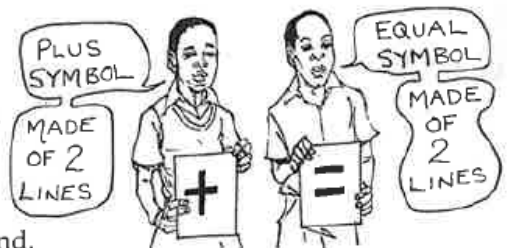
Activity 2 Writing addition sentences

Time allocation: 3 lessons

Suggested resources
bottle tops, stones, leaves, strings, fingers, number cards

Instructions

- 1 Have two sets of objects of the same kind.
- 2 Ask the learners to add any 2 given sets of objects.
- 3 Let them say the addition process in their local language.
- 4 Tell the learners that the symbol for plus is + and equals is =
- 5 Place cards with symbols representing number of objects below them as follow:




 $2 + 1 = 3$

Figure 4-8: Suggestions for introducing addition from the Teachers' Guide for Standard 1 (Source: Malawi Institute of Education, 2012b, p. 20).

Figure 4-8 shows the first 5 instructions written in the teachers' guide for the class activities observed during Lesson 1. The remaining instructions, not shown in Figure 4-8, require the teacher to demonstrate how to write addition statements such as $2 + 1 = 3$, then let the learners practice writing the + and = signs in the air, on the ground, as well as in their notebooks. The teacher achieved the same goals without necessarily going by the instructions in the same manner as spelt out in the teachers' guide. For example, instruction 5 in Figure 4-8 asks the teacher to place number cards below physical objects given to groups during the lesson.

Instead, the teacher drew sketches of the objects (stones, sticks, and leaves) on the chalkboard and wrote the numbers below the drawings (see Figure 4-18). Likewise, instead of demonstrating how to write $2 + 1 = 3$, the teacher built on learners' ideas and guided them to come up with the written statement themselves (see Figure 4-32).

During an unstructured interview, the teacher mentioned that she rarely selected examples from the learners' textbook. Instead, she opted to formulate the examples by herself. The teacher explained that she preferred not to use the learners' textbook because some of the illustrations were potentially confusing to the learners. A case in point was the illustration in Figure 4-9 showing an open book with vivid scaled drawings of a tree, a house and a flower. Since the illustration of the book was provided alongside other illustrations, asking learners to state how many things they see in each box, it would be potentially challenging for them to see the box as having one book or else be attracted to the other vivid items on the page (see Figure 4-9).

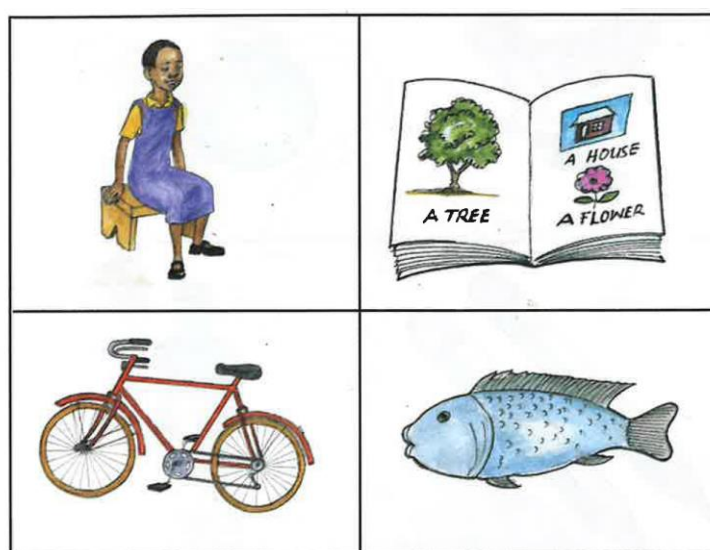


Figure 4-9: A textbook task requiring learners to count the things they see in a box (Source: Malawi Institute of Education, 2012a, p. 1).

One of the key features of the observed tasks was their considerably long duration, which affected the lesson time, as mentioned earlier. In an interview, the teacher indicated that

marking learners' notebooks took a lot of lesson time because of class size and the nature of the learners. Instead of collecting the notebooks to mark later, the teacher found it easier to ask learners to open page to be marked. Despite seemingly consuming much of the lesson time, the teacher considered it necessary to give individual tasks and mark them during the lesson. The teacher indicated that she also used the marking session to know more about her learners and offer them one-to-one assistance.

Regarding the number of problems given during individually done tasks, the teacher said during an interview that she did not give more than three problems. Giving more problems would mean a lot of marking when the number of problems is multiplied by the number of learners. For instance, during Lesson 2, the teacher gave a single example to the 170 learners who were present that day as homework to be marked the next day (see Figure 4-10). Still, the teacher told some learners who repeatedly failed the individual work despite her support, to do the failed problems at home and show the teacher the next day.

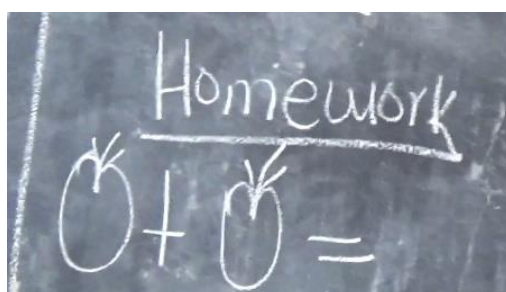


Figure 4-10: A homework task in Lesson 2 of Standard 1 (Source: Researcher).

Even though mathematics was taught every day of the week, the teacher explained that she gave homework twice in a week. The teacher indicated that homework tasks offered an opportunity for learners to seek support from parents and relatives at home. During marking, the teacher was heard telling some learners: "Please write again at home and show me tomorrow morning. Mum should help you!"

Sometimes, learners whose work was marked at the beginning would have nothing to do and would start making noise as the teacher was marking the work of the rest of the class. In such scenarios, the teacher was also prompted to give them homework, so that those who had finished their work earlier would start working on it and be occupied.

The rationale for the teacher's selection of examples

Regarding the selection of examples, the teacher indicated in Excerpt 4-3 that there was no particular pattern or order that was followed, as long as the addends were not exceeding 5.

190. T: Yeah. We just select. Maybe I should put here two plus zero or two plus two. Make sure that it should not exceed five.

Excerpt 4-3: Selection of examples

4.3.3 Mediating artefacts

The teacher worked with various types of artefacts across the six lessons. The rationale for the teacher's selection of artefacts has been discussed towards the end of this section.

Nature of artefacts

The artefacts used across the Standard 1 lessons have been presented in Table 4-10 that follows.

Table 4-10: Artefacts used across the six Standard 1 lessons

Lesson	1	2	3	4	5	6
Artefacts	Books, stones, leaves	Sticks, leaves	Prewritten papers	Framed counters, leaves, prewritten papers	Framed counters, prewritten papers	Framed counters, fingers, prewritten papers

During Lesson 1, conducted at the beginning of the school day, the teacher allocated books, stones, leaves, and sticks to each group (see Figure 4-11). However, only the books, leaves, and stones were used in Lesson 1, while the sticks were used during Lesson 2 that was carried out later towards the end of the school day. During Lesson 4, the learners were initially asked

to use leaves, but the lesson proceeded with the use of their hand-made framed counters. In Lessons 4 to 6, the teacher distributed small prewritten papers to groups. She also used prewritten chart-size papers pasted on the chalkboard, such as the paper shown in Figure 4-37.

The Standard 1 teacher's use of artefacts

Table 4-5 and Table 4-10 show the significant role of artefacts during the six lessons. The teacher used them for presenting tasks, finding solutions to the given tasks, and managing the class.

Use of objects from the surroundings

During the first lesson, physical artefacts from the nearby surroundings (books, stones, and leaves) were used for presenting tasks, such as: “Find two books plus one book.” During the first two lessons, the artefacts to be used were placed in groups before the commencement of the lesson. This made it possible to present the addition problems by directly referring to the items at hand among the learners (see Figure 4-11). The use of objects they saw every day possibly made the introduction of addition fit the learners' familiar context.



Figure 4-11: Books, stones, leaves, and sticks used in Lessons 1 and 2 of Standard 1 (Source: Researcher).

The learners were supplied with more items than would be required during the calculations. The teacher explained which items of a particular type should be counted and which ones should not be counted.

The teacher also seemed to use the physical artefacts for ensuring learner engagement. Even during group activities, the teacher observed if everyone in the group was following the counting (see Utterance 130 of Excerpt 4-4).

126. T: Now let us pick three sticks. How many sticks?
127. C: Three.
128. T: Let us count three!
129. C: One, two, three!
130. T: Everyone should be counting. [<i>Reprimands L23</i>] You! Nelson! Count with your friends!

Excerpt 4-4: Learner encouraged to participate during counting objects activity.

Use of framed counters

The teacher started using framed counters for calculations from Lesson 4 (see Figure 4-12), where she used them alongside leaves. During Lessons 5 and 6, framed counters were the sole artefacts used for working out all the tasks.

The teacher appears to have informed the learners to make their own framed counters because they were observed carrying them during Lesson 1. During Lesson 2, the teacher reminded learners to bring counters the next day (see Excerpt 4-5).

509. T: I said tomorrow when coming you should bring what?
510. C: Counters!
511. T: What should you bring with you tomorrow?
512. C: Counters!

Excerpt 4-5: Teacher reminding learners to bring personal counters to class.

Before switching to exclusive use of framed counters for working out all the tasks from Lesson 5, the teacher also reminded the learners during Lesson 4 to bring their framed counters the next day.

It was observed that in addition to finding solutions, the teacher also used learners' framed counters for ensuring active learner engagement. She was able to note those who were participating or not. To achieve this, the teacher taught the learners the appropriate posture for holding counters during Lesson 4—leaving one hand free for sliding the counters as one holds the frame (see Figure 4-12). Learners were observed working with their counters exactly in the same manner as the teacher did.



Figure 4-12: Teacher demonstrating how to hold framed counters (Source: Researcher).

The teacher and the class counted all the counters when working out the sum. The use of the count-all strategy will be discussed under the teacher's "mediating talk and gesture for providing methods for generating solutions" in section 4.3.4.

Use of prewritten papers

Starting from Lesson 3, prewritten papers were used for presenting tasks to be completed by learners, by appending the answer to the right-hand side of the equal sign. During Lesson 3, the teacher used prewritten drawings on chart-papers pasted on the chalkboard (see Figure 4-13).

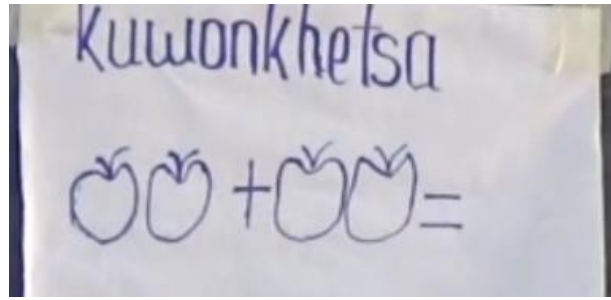


Figure 4-13: A prewritten task on chart-paper in Lesson 3 (Source: Researcher).

During Lessons 4 to 6, the teacher presented tasks to be done in groups using prewritten pieces of paper, which required learners to complete by writing the answer. After the answers were found and written down by the groups, they were pasted on the chalkboard for verification by the whole class. The teacher asked representatives from the groups to read their pasted papers (see Figure 4-14). This was followed by the whole class reading the written statement again. Thereafter, the teacher worked out the solutions with the class to decide whether to maintain the written answer or to change it. When there was a need to change, another member of the group that initially solved it was asked to come in front and write the corrected answer. In some cases, this would require learners making several attempts before the correct answer was given, ultimately affecting the final appearance of the chart (see Figure 4-26 in section 4.3.4).

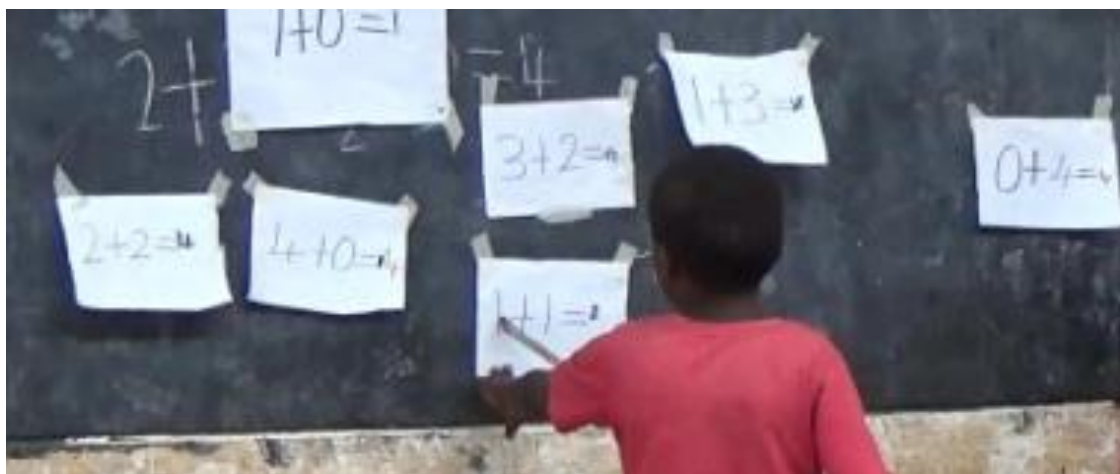


Figure 4-14: A group representative reading out their work (Source: Researcher).

At the end of the lesson, the papers were transferred from the chalkboard to the walls of the classroom, and they could still be seen in the subsequent days. During Lesson 6, papers from all the previous lessons were still visible on the classroom wall.

Use of fingers

Learners appeared to be familiar with the use of fingers when working with numbers. The first observation with their use of fingers was made while singing the number song during Lesson 1. The learners followed the teacher's fingers when mentioning the numbers during the song (see Figure 4-15).



Figure 4-15: Using fingers when counting numbers (Source: Researcher).

During Lesson 6, the teacher asked those who did not have their counters to use their fingers as shown in Excerpt 4-6, Utterance 893.

- | |
|---|
| 891. T: Let us count one! |
| 892. C: <i>[Count as the teacher pushes a counter on the frame]</i> one! |
| 893. T: Those of you without counters, begin! Use your hands! Use your fingers! |
| 894. C: <i>[Some learners raising their hands showing an index finger]</i> one! |
| 895. T: one! Alright? |
| 896. C: Yes! |

Excerpt 4-6: Teacher asking learners without counters to use their fingers.

During the lessons, some learners were observed counting using fingers as the teacher was using counters. As it can be seen in Figure 4-16 that follows, some learners used fingers during group work even when counters could be seen nearby.



Figure 4-16: Learners using fingers during calculations (Source: Researcher).

Overall, the teacher's use of artefacts was structured. The teacher achieved systematic fading of artefacts from one lesson to the next (Venkat & Askew, 2018). As shown in Table 4-10, the teacher used them for presenting tasks during the first two lessons. By the third lesson, the teacher had shifted to the use of unstructured inscriptions (drawings) for presenting the tasks. From the fourth lesson onwards, the teacher entirely shifted to the use of structured mathematical statements for presenting tasks during the lessons.

The rationale for the Standard 1 teacher's use of artefacts

A key factor guiding the teacher's choice of the observed artefacts was availability. The teacher used artefacts that were readily available from the surrounding environment and kept on reminding learners to make their counters as indicated in Excerpt 4-5. During an interview, the teacher stated that she would just easily ask the learners to go outside the classroom and pick leaves, sticks, or stones, which she would use in her lessons. However, she indicated that doing this takes time, and learners may start playing outside. As such, the teacher resorted to the use of the hand-made framed counters as explained in Excerpt 4-7.

195. R: So, when you are planning, you also think about what resources you use, the artefacts. How do you select these?
196. T: Ah, to say the truth, *[laughs]* we just select anyhow. There's no..., aah, I can say that... I take these from, maybe, teachers' guide? No. But what we did is, we make sure that we can take everything because they say, aah, learners should learn in the local environment. So, we just go there and look for what? Local environment. It's not a matter of going to the shop and buy. No. No. But just send the learners outside: 'Can you go there and get stones?' 'Can you go there and get leaves?' 'Can you go there and get sticks?' So, when planning you can choose: 'Today I am going to use leaves'; okay, 'today I am going to use what?...' But, aah, sometimes, maybe I can say often, we use counters for not consuming the time. Because when you send the learners outside maybe others will start playing. That's why we just say: okay, make what? Counters.

Excerpt 4-7: Teacher explaining how she selects the artefacts.

The teacher commented that the use of objects from the surroundings requires no cost. The teacher's selection and use of artefacts was as outlined in the teacher's guide (see Figure 4-8). As regards the use of prewritten papers, the teacher indicated during an interview that it was important to familiarise learners with writing on all sorts of materials, including chart-papers regardless of how the final output would look like after their repeated attempts.

4.3.4 Mediating inscriptions

The teacher used chalkboard inscriptions for presenting tasks that were to be done during the lessons, alongside the prewritten papers discussed in the preceding section (4.3.3). Usually, the teacher asked the learners to come forward and present their offers in writing.

Nature of inscriptions in Standard 1

As mentioned in section 4.3.2, the first task in Lesson 1 required learners to write the + and = signs in the air. The teacher taught the writing of addition and equal signs independently followed by a discussion of how they are used in an addition statement. Section 4.3.2 explains how the teacher's approach slightly differed with the suggestions from the teachers' guide, shown in Figure 4-8. The affordances of the teacher's approach have been discussed under the teachers' "mediating talk and gesture for advancing learning connections" under section 4.3.4.

In Excerpt 4-8 that follows, the teacher explained how Standard 1 learners are normally taught how to write:

214. T: How to write? We have several ways. Aah, first, we start to write in the air.
215. R. Okay?
216. T: If you had come when I was teaching numbers you could see that. Because when we say: “Let’s write four!” We say: “Dot! Then down! Then right! Then....” Those things. We first start in the air, then after in the air, it’s when we go on the ground, before they write in the exercise book.

Excerpt 4-8: Teacher explaining procedure for teaching how to write.

The teacher stated in Excerpt 4-8 that before learners could start writing in their notebooks, they usually had to write in the air, followed by writing on the ground outside the classroom. The teacher continued to say that she has to mark what they write on the ground, even if there would be 200 learners present on that day. The ones marked correct on the ground are then told to go in the classroom and write in their notebooks. During the six lessons, all the writing tasks were done in the classroom. The teacher also taught the learners how to write in their notebooks (see Figure 4-17). She explained the use of the notebook margin as well as the space on the notebook page.



Figure 4-17: Teacher demonstrating how to use a notebook (Source: Researcher).

The Standard 1 teacher’s use of inscriptions

During the first two lessons on addition, the teacher started using inscriptions as visual representations of tasks given verbally. During Lesson 2, for instance, learners were initially

asked to find “two sticks plus three sticks”. After finding the answer, the teacher started by drawing sticks as a way of recording the statement “two sticks plus three sticks equals five sticks”. After this, numbers were introduced below the sticks to come up with a symbolic representation of the same statement, that is, “ $2 + 3 = 5$ ” (see Figure 4-18).

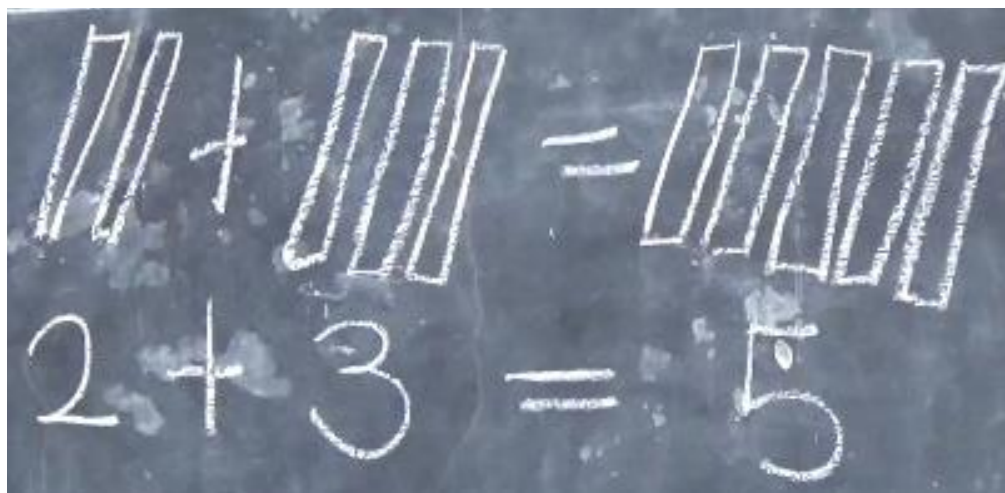


Figure 4-18: Teacher's representation of “two sticks plus three sticks equals five sticks” (Source: Researcher).

After demonstrating the association between numbers and drawings shown in Figure 4-18, the teacher started using drawings for presenting tasks. Figure 4-19 shows the individual work that was given to learners during Lesson 2 using drawings.



Figure 4-19: Individual work presented using drawings in Lesson 2 (Source: Researcher).

After presenting the task shown in Figure 4-19, learners were asked to “read” the visual representation of the task before working out the solutions. She also repeatedly informed the learners to draw the total number of items before writing the number representing their sum.

However, despite the teacher's repeated explanations, most learners kept on just writing the numerical value of their answer (see Figure 4-20).

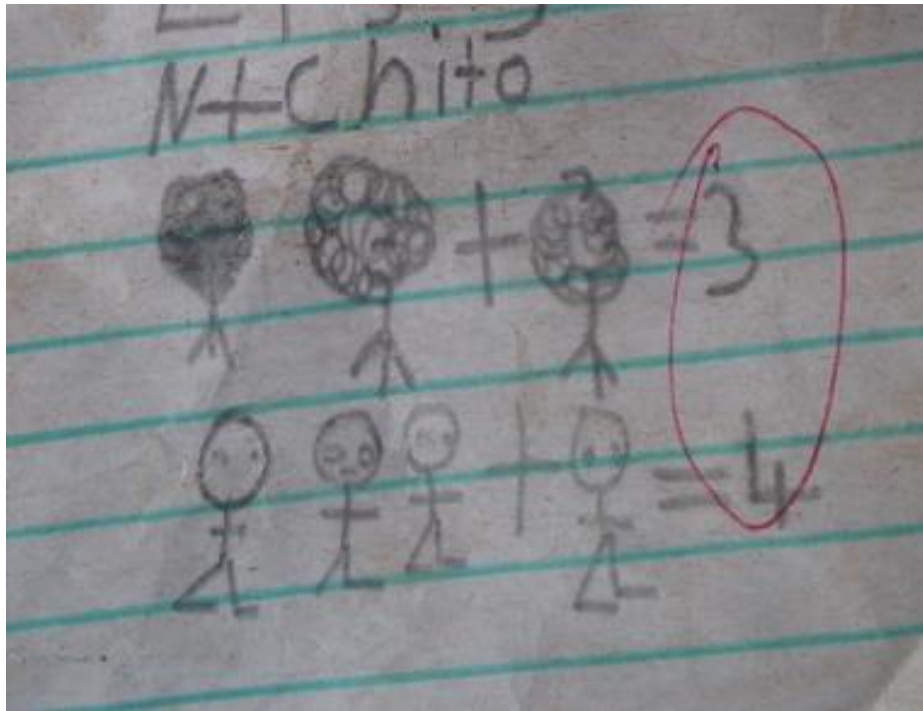


Figure 4-20: Learners writing a numerical answer when drawings were expected (Source: Researcher).

While marking the learners' work shown in Figure 4-19, the teacher kept on reminding them to draw first before writing the numbers. However, some learners kept coming back to the teacher with the presentation of the answer shown in Figure 4-20 over-and-over again.

The use of drawings might not have been easier for some learners. During lesson 2, for instance, one learner was observed being more artistic with the drawings of people, thereby taking more time, instead of just using stick diagrams as demonstrated by the teacher. Some learners, however, followed the teacher's instructions and the teacher marked as correct (see Figure 4-21).

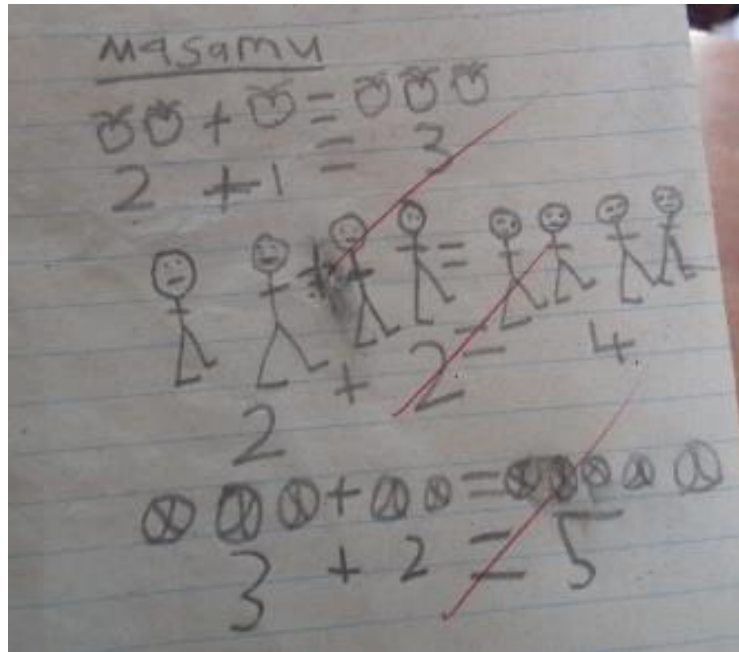


Figure 4-21: Learner's drawings of object with numerical representations as required by the teacher (Source: Researcher).

When reviewing previous learning during Lesson 4, it was noted that some learners had not yet adopted the teacher's method for generating solutions to addition problems given in the form of drawings (see Figure 4-22).

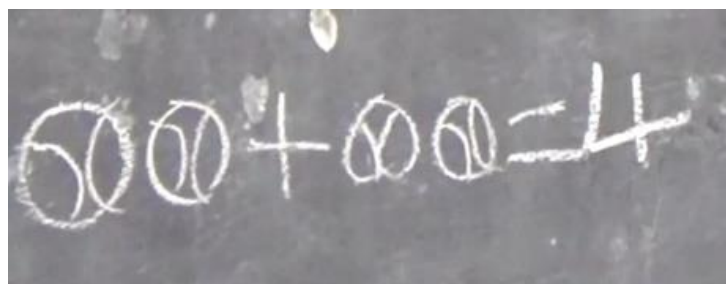


Figure 4-22: Numerical answer appended to drawings (Source: Researcher).

After noting the answer given by the learner in Figure 4-22, the teacher explained again why it was necessary to include the drawings, stating that the problem was presented using balls, hence the answer should also show balls. After explaining the rationale for including drawings during Lesson 4, the teacher informed learners that the nature of tasks to be done next would

no longer require making drawings. The teacher's use of inscriptions had now shifted from the use of drawings during the first three lessons to the sole use of structured mathematical statements during the last three lessons (see Figure 4-23).

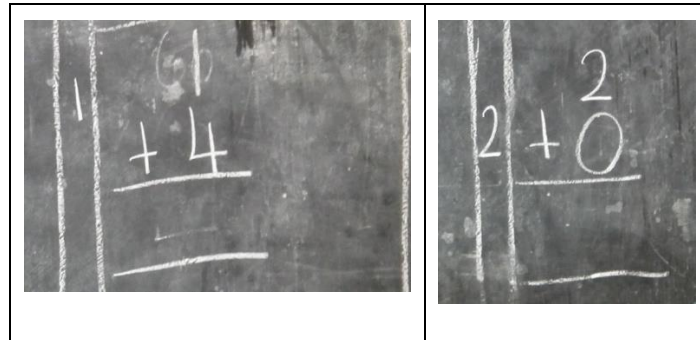


Figure 4-23: Individual work presented using structured statements in Lesson 6 (Source: Researcher).

For numbers written by learners, the teacher was not only interested in whether the value was correct and readable, but also checked the presentation of the number (see Figure 4-24). The teacher asked the class to check if the 2 given in Figure 4-24 was correctly written and how it would be improved.

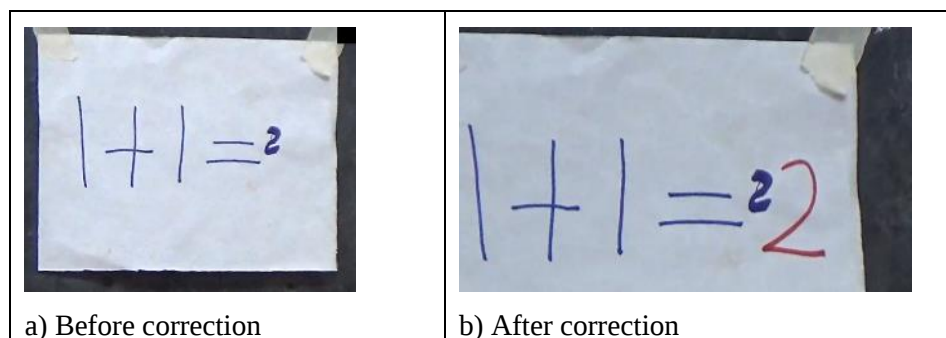


Figure 4-24: Correcting the shape of a written number (Source: Researcher).

The way the teacher handled learners' errors associated with writing has been discussed further under the teacher's "mediating talk and gesture for advancing learning connections" in section 4.3.4.

The rationale for the Standard 1 teacher's use of inscriptions

The teacher indicated that the preliminary stages of writing in the air and on the ground are done to prepare the learners before they could write in their notebooks. The inscriptions of stick drawings seemed to be utilised for modelling the process of addition. It provided a way of letting the learners show how they arrived at the numerical sum of the drawn items.

During Lesson 6, the teacher also emphasized the need to align the numbers during vertical addition. This skill would be required when dealing with place-value alignment of digits in later classes. Most of the writing across the six lessons was done on prewritten papers.

4.3.5 Mediating talk and gesture

This section presents the observations made on the Standard 1 teacher's use of mediating talk and gesture for providing methods for generating solutions, building mathematical connections, as well as advancing learning connections.

Mediating talk and gesture for providing methods for generating solutions

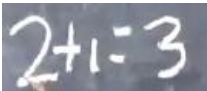
Even though all the tasks across the six lessons involved finding the sum of two numbers with a sum not exceeding 5, the steps followed varied depending on the way the task was presented (see Table 4-5) and how the answer was worked out (see Table 4-10). There were similarities and overlaps in the methods used as the teacher progressed between Lessons 1 and 2, 2 and 3, and from 3 to 6.

Steps followed during the first two lessons

The approach used for finding solutions to problems during the first two lessons has been summarised in

Table 4-11 that follows. The actions were all done by learners in their groups while following instructions from the teacher.

Table 4-11: Methods for generating solutions during Lesson 1 and Lesson 2

Stage	Description	Example
Problem Presentation	Verbal reference to some given artefacts	Add two books and one book
Steps	Count the first number of items	Count two books
	Count the second number of items	Count one book
	Count all the items to get the sum	Count three books
	State the resulting addition statement	Two plus one equals three
	Write the resulting statement	

As pointed out under the teacher's use of inscriptions in section 4.3.4, towards the end of Lesson 1, the teacher introduced an intermediary stage of drawing the items being worked upon, followed by writing the numbers below the items (see Figure 4-25). The leaves drawn in Figure 4-25 represented physical leaves that had earlier been used during the presentation of the problem. Before they were drawn on the chalkboard, the leaves had been physically manipulated when finding the required answer, as was the case with the books in

Table 4-11.

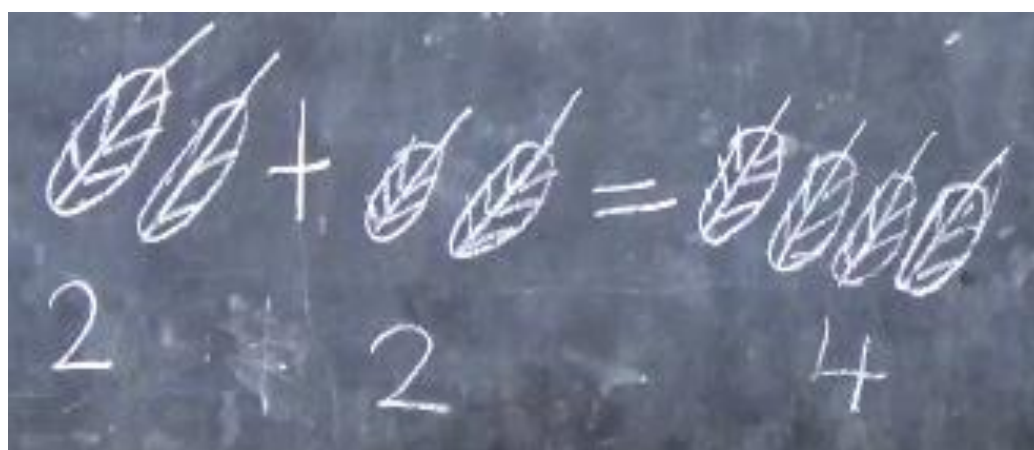


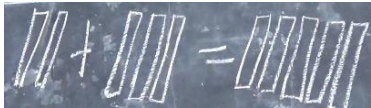
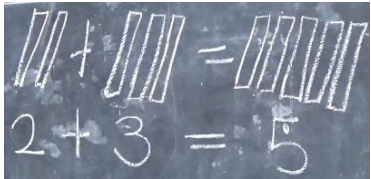
Figure 4-25: Visual representation of “2 leaves plus 2 leaves equals 4 leaves” (Source: Researcher).

A similar representation was later used in Lesson 2 to represent the sum of 2 sticks and 3 sticks (see Figure 4-18).

Steps followed during the second and third lessons

The use of drawings provided another mode for presenting problems during Lesson 2 and Lesson 3, without necessarily referring to physical artefacts that were present. During Lesson 2, drawings were used for presenting individual work (see Figure 4-19) and homework (see Figure 4-13). The shift in the method for working out solutions to problems from Lesson 2 to Lesson 3 has been presented in Table 4-12.

Table 4-12: Methods for generating solutions during Lesson 2 and Lesson 3

Stage	Description	Example
Problem Presentation	Visual representation of the given problem	
Steps	Count the first number of drawn items	Count two sticks
	Count the second number of drawn items	Count three sticks
	Count all the drawn items to get the sum	Count five sticks
	State the resulting addition statement	"2 plus 3 equals 5"
	Draw the total number of items	
	Write the number of items below the drawings	

Apart from the procedure laid out in Table 4-12, the teacher also exposed the learners to quick mental problem-solving.

Steps followed during the last three lessons

For Lessons 4 to 6, both the presentation of problems and the steps for finding the solution were adjusted slightly. The problems to be solved were now presented as addition statements.

The solution was found using learners' personal framed counters. The steps carried out during Lessons 4 to 6 have been presented in Table 4-13.

Table 4-13: Methods for generating solutions during Lessons 4 to 6

Stage	Description	Example
Problem Presentation	Pastes a chart on the chalkboard	
Steps	A learner reads the written statement	"One plus three equals"
	Class verifies the reading and reads the statement again	"One plus three equals"
	A learner writes a suggested answer	4
	Class verifies with counters: Counts the first addend	Count 1
	Counts the second addend	Count 3
	Adds the two sets of counters	Add 3 counters to 1 counter
	Reads the resulting addition statement	"one plus three equals four"

If the answer proposed by the learner who wrote on the paper was wrong, another learner was asked to overwrite it with the correct answer using a different coloured marker. For vertical addition statements solved during Lesson 6, the teacher lastly checked on the alignment of the written answer with respect to the given addends (see Figure 4-26).

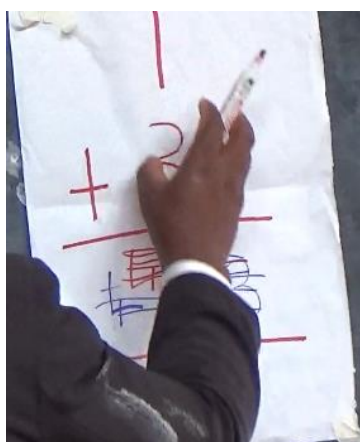


Figure 4-26: Emphasizing the alignment of the answer and the addends (Source: Researcher).

In Figure 4-26, there were correct answers that were initially written by the learners (see Figure 4-37), but they were crossed out by the teacher because they were not properly aligned with the other addends. In some cases, the given answer would be correct, but the shape would be incorrect. In that case, the teacher provided a method for justifying the correctness of the given inscription as discussed under the teacher's "mediating talk and gesture for advancing learning connections".

During Lessons 4 to 6, the correctness of the answers was checked using framed counters. In some cases, learners could not tell that the answer offered by their classmates was wrong until they later checked with the teacher using counters. The learners were taught from Lesson 1, not to intuitively give answers before they had physically counted, as stated in Utterances 420 and 422 in Excerpt 4-9.

418. T: ...How many stones have we picked?
419. C: Four!
420. T: We did not count. We did not do what?
421. L32: We did not count!
422. T: How have you known that it is four? So, I want you to pick them one by one and count with your friends. Alright?
423. C: Yes!
424. T: You should be picking what? One by one and count with your friends. Let us begin. 1!
425. C: <i>[Counting]</i> One, two, three, four!

Excerpt 4-9: Teacher instructing learners to count objects one by one.

Even when working with 0 as an addend, the teacher also expected the class to count that 0 and add it to the other addend. For instance, when working out $2 + 0$ with the class during Lesson 4, the teacher remained consistent with the requirement for counting (see Utterance 303 in Excerpt 4-10).

297. T: Thereafter, we expect it to count how many more? [*Pointing at 0*]

301. T: Let us count zero, alright?

302. C: Yes!

303. T: Let us count that zero! Let us count!

304. C: Zero!

Excerpt 4-10: Expectation to count zero.

The learners thought that just by mentioning “zero” they had done the expected counting, but the teacher asked them to physically count the zero as was the case with other addends dealt with all along. The teacher demonstrated the addition of zero and two on the framed counters by sliding fingers along an empty string towards the previously counted two counters. This enabled learners to make a visual connection between the various representations of zero.

Even though the just discussed teacher’s connections when working with zero were extended from her methods for generating solutions, they also signify more on the teacher’s “mediating talk and gesture for building mathematical connections” discussed in the next sub-section.

Mediating talk and gesture for building mathematical connections

During the six lessons, the teacher made several connections between mathematical content, and also connected various means of mediation. The teacher made several attempts to make strong connections within examples as illustrated in the way she worked with 0 in Excerpt 4-10.

Connecting various means of mediation

The teacher made strong connections between artefacts, inscriptions, talk and gesture. In Figure 4-27, the teacher simultaneously pointed at the numeral 1 as it was being mentioned, and shortly represented it using a counter.



Figure 4-27: Linking various means of mediation (Source: Researcher).

When reading inscriptions on the chalkboard, the teacher always pointed to each part of the inscription being mentioned (see Figure 4-28).

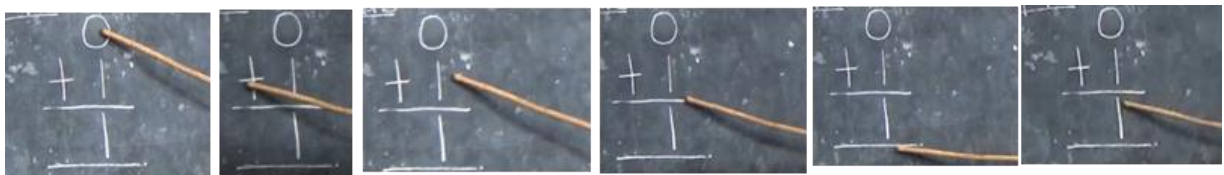


Figure 4-28: Pointing at each part of a written inscription (Source: Researcher).

To train the learners to notice her use of a pointer, the teacher would freeze the pointer at one point, and observe if the class would continue reading the next parts that have not been pointed to yet.

Use of contrast

In some instances, the teacher emphasized on both what something was and what it was not. While teaching the correct alignment of numbers during vertical addition, the teacher explained what the correct way of writing was and what was not correct (see Figure 4-29).



Figure 4-29: Demonstrating correct and incorrect alignment (Source: Researcher).

The teacher also used contrast when highlighting sources of learners' errors. This has been discussed under the sub-section: "Mediating talk and gesture for advancing learning connections".

Use of gesture

The teacher used hand gestures related to some concepts being explained. The hand gestures made it possible for the learners to connect the verbal and written representations of the discussed concepts with their visual conceptualisations used in everyday communication. For instance, when referring to zero, the teacher used empty hands gesture that corresponded to the mention of "nothing" (see Figure 4-30).



Figure 4-30: Gestures for "nothing" when referring to zero (Source: Researcher).

To reinforce the meaning of zero, the teacher also utilised routine practices such as handclaps to evoke their thinking. After asking the learners to clap once during Lesson 4, as they usually do, the teacher asked them to clap zero times (see Excerpt 4-11).

- | | |
|-----------|---------------------------------------|
| 408. | T: Raise your hands up! |
| | |
| 412. T: | ... Up! Clap once! |
| 413. C: | [Clap once] one! |
| 414. T: | Clap zero times! |
| 415. C: | [Clap once] |
| 416. T: | Eeh! [Laughs] Ooh! Clap zero times! |
| 417. C: | [A few clap once, others do not clap] |
| 418. C+T: | Aah! [Laugh] |

Excerpt 4-11: Clapping zero times.

Similar to what was done with handclaps in Excerpt 4-11 from Lesson 4, the teacher also started Lesson 5 with handclapping, asking the learners to clap from four times down to once.

When introducing addition in Lesson 1, the teacher joined her palms together every time she referred to “adding” or “together”, as shown in Figure 4-31.



Figure 4-31: Gestures for “adding” or “together” (Source: Researcher).

On the chalkboard, the + sign was sometimes emphasized by tracing it with a pointer when mentioning “plus”. During Lesson 1, the teacher enabled learners to connect the + sign with the verbalised movement of the hand when writing it: “Dot! Down! Cut-in-the-middle!” This hand movement for the + sign was used when trying to explain why a + sign that was written by a learner during Lesson 6 was not accepted by the class. This has been discussed under the teacher’s “mediating talk and gesture for advancing learning connections” in the next sub-section). After verbalising the hand movement of the + sign, the teacher also asked learners to think about their own way of verbalising the hand movement of the = sign. This technique of working with learners’ ideas has been discussed in the next sub-section.

Use of language

During the lessons, both the teacher and learners mentioned numbers in both English and Chichewa. For instance, during Lesson 1, when the teacher asked the learners the number of books they were working with, the learners mentioned “two” for two books, but shortly thereafter, they said, *limodzi* [one] for one book. Even though both the Chichewa and English names referred to the same number, say 1, it was possible to accept both “limodzi” and “one” from the learners as the numeral 1. However, as shown in Excerpt 4-12, it seems the teacher

wanted to maintain consistency in the naming of the numbers. The teacher asked the class in Utterance 211 of Excerpt 4-12 to call the numbers with their English names by asking what they called it as a number.

203. T:	Aha! Those ones. Let us start: How many books did we pick at first? [<i>Showing two books in the left hand</i>].
204. C+T:	Two!
205. T:	How many books did we pick? [<i>Showing two books in the left hand</i>].
206. C:	Two!
207. T:	After that, we picked how many books? [<i>Showing one book in the right hand</i>]
208. C:	One! [<i>Said the Chichewa word “limodzi” for 1 with the appropriate prefix “li-”</i>]
209. T:	Eh? [<i>Showing one book in the right hand</i>]
210. C:	One. [<i>Repeated the Chichewa word for 1</i>]
211. T:	One, what do we call it as a number?
212. C:	One.

Excerpt 4-12: Naming numbers in English and Chichewa.

When the learners were asked how many physical objects they could see, during the first three lessons, there were more chances of mentioning the quantity in Chichewa, yet a written numeral was always called by its English name. The reference to numbers using both Chichewa and English gradually faded when problem-posing during the later lessons shifted from the use of physical artefacts to written numbers during the last three lessons. In the last three lessons, the numbers in the structured addition statements were always called by their English names, while the + and = signs were always referred to using their Chichewa names.

Mediating talk and gesture for advancing learning connections

There were several aspects of the Standard 1 teacher’s mediating talk for advancing learning connections that were observed.

Working with learners' ideas

The teacher tried to work with the learners' thinking and guided them through until they came up with the expected solution. After teaching the learners how to write the + and = signs, the teacher asked the learners to write down the statement "two books plus one book equals three books". As the learners started writing, the teacher noticed their thinking and changed her emphasis until the correct statement was written by the 10th learner (see Figure 4-32).



Figure 4-32: Guiding learners through their thinking (Source: Researcher).

As the first five learners wrote "3", "+ = ", "3", "= 3", and "+ = 3 1" shown in Figure 4-32, the teacher noticed that they were only focusing on one aspect of the given statement, mostly what was said last (the answer). The teacher then changed her approach and started voicing out the statement as the learner approached the chalkboard and started writing. This made the sixth learner write "2 1 + 3" thereby including most of what was said by the teacher. At this point, the teacher decided to start asking the class to read what was written by the learners and check against the given statement. The class read out the learner's written statement following the teacher's hand that pointed at each element of the written statement on the chalkboard (see Figure 4-33).

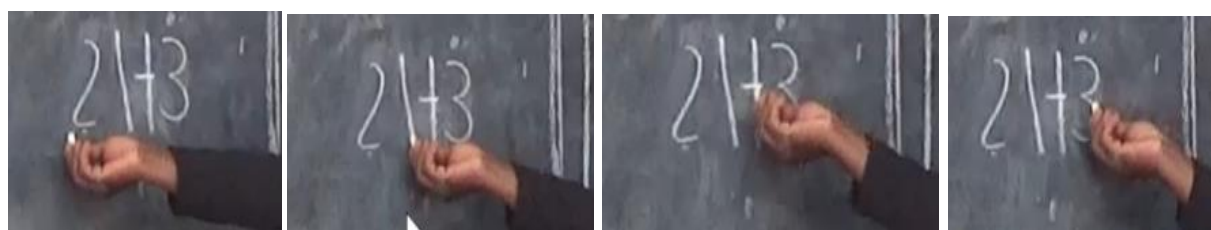


Figure 4-33: Checking a statement written by a learner (Source: Researcher).

After the teacher started discussing with the class why the offered solution was not correct, the last three learners (see Figure 4-32) started getting closer and closer to the correct answer. The 10th learner gave the correct answer.

During Lesson 6, the teacher used the same approach when introducing vertical addition, letting the learners try out writing the statement. The teacher focused her discussion on what she called the differences in the equal signs. The teacher informed the learners that the new equal sign to be used on this day was basically an extension of the usual one that had been used up to this day (see Figure 4-34).



Figure 4-34: Teacher's presentation of "usual" and "new" equal signs (Source: Researcher).

The teacher and the class compared and contrasted the two equal signs shown in Figure 4-34 by examining the length of the drawn lines, the space between them and where the answer is placed. After the discussion about the signs, two learners attempted to write the statement "0 plus 1 equals" on the chalkboard using the new notation—where "the answer is placed inside the equal sign rather than to the right of the sign". The learners appear to have attempted to include the answer in the notation as described by the teacher (see Figure 4-35).



Figure 4-35: Learners' attempt to apply the teacher's description (Source: Researcher).

The teacher came in and wrote the expected notation shown in Figure 4-36. Thereafter, the teacher emphasised that the addends had to be aligned vertically.

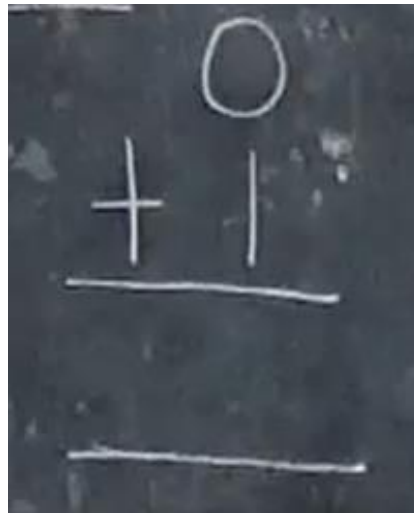


Figure 4-36: Expected notation written by the teacher (Source: Researcher).

Considering the way the teacher introduced the writing of both horizontal and vertical addition statements in Figure 4-33 and Figure 4-35, the teacher was able to reach to a new approach or concept by patiently building on the ideas of learners.

Verification of errors

When verifying offers from learners, the teacher approached the errors in stages, isolating one error at a time until all the issues were resolved. At each stage, the teacher discussed with learners the logical argument for establishing the error. During Lesson 6, for instance, the

teacher invited learners to come to the chalkboard to write the answer to $\begin{array}{r} 1 \\ + 3 \\ \hline \end{array}$. Three learners

took turns writing the answers as 5, 3, and 4. After the third one had written a 4, the learners stopped raising their hands, but the teacher asked if there was any other learner who might have a different answer. The fourth learner repeated writing 4 as shown in Figure 4-37.

$$\begin{array}{r} 1 \\ + 3 \\ \hline 4\ 4\ 5\ 3 \\ \hline \end{array}$$

Figure 4-37: Offers by four learners (Source: Researcher).

To show the learners why the resulting written structure was not correct, first, the teacher invited the class to read the answer shown in Figure 4-37. The class read the answer as “one plus three equals three, five, four, four” as the teacher pointed to each digit or symbol. By letting the class read it as “one plus three equals three, five, four, four”, the answer probably sounded unusual to the learners, which they unanimously rejected. Secondly, the teacher worked out the correct answer with the class. After working out the correct answer with counters, the teacher then discussed the placement of the answer with respect to the given addends (see Figure 4-26).

In some cases, the answer would be wrong, but the teacher was not quick to dismiss the wrong answer and work on the correct one if there was something else to be learnt from the wrong answer. During Lesson 5, one group wrote the answer to their paper as shown in Figure 4-38.

$$3 + 2 = 4$$

Figure 4-38: A wrong answer that was written as flipped 4 (Source: Researcher).

The representative of the group that wrote the answer read the statement in Figure 4-38 as “three plus two answer four”. The class was divided on the way the 4 in Figure 4-38 was written. The teacher corrected the error in Figure 4-38 by asking another learner to come and write 4 on the chalkboard (see Figure 4-39).



Figure 4-39: A learner writing 4 correctly (Source: Researcher).

It can be seen in Figure 4-38 and Figure 4-39 that the teacher decided to firstly remediate the writing of 4 before checking if the sum of 3 and 2 is indeed 4. The teacher employed similarity and contrast to help to clear the difficulties that some learners had with the correct orientation of 4, as discussed in the next sub-section.

Using similarity and contrast to remediate errors

After the learner shown in Figure 4-39 had written a 4, the teacher asked the class if the just written 4 was correct. Rather than basing on sentimentality, or common logic, the teacher provided a method for justifying the correctness of the 4 using its corresponding hand movement: “Dot! Down! Turn-right! Cut-in-the-middle!”, showing the movements with a pointing stick. The teacher continued the discussion of how to write 4, by emphasizing what is 4 in contrast to what is not 4. The teacher wrote another 4 while verbalising the correct hand movement (see Figure 4-40). The other 4 was written above the one written by the learner.

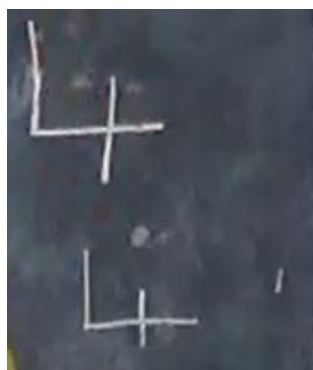


Figure 4-40: A similar correct 4 (Source: Researcher).

The teacher continued the discussion by discussing what is not 4. The teacher verbalised the hand movement of what is not 4 (“Dot! Down! Turn-left! Cut-in-the-middle”) and simultaneously wrote down the wrong 4 adjacent to the two correct 4s as shown in Figure 4-41.



Figure 4-41: A contrasting a wrong 4 with two correct 4s (Source: Researcher).

The discussion of writing 4 ended by emphasising on the key aspect of the hand movement that the teacher noticed as the main challenge among the learners, that is, the horizontal direction of the hand. The teacher then took back the class to Figure 4-38, to check if 4 was the correct answer to $3 + 2$.

The teacher’s effort in remediating the writing of 4 helped to spend less time in the discussion that followed soon after working out the correct answer for $3 + 2$. The next group of learners had worked out $1 + 3$ and correctly found the answer as 4 that was written as shown in Figure 4-42.

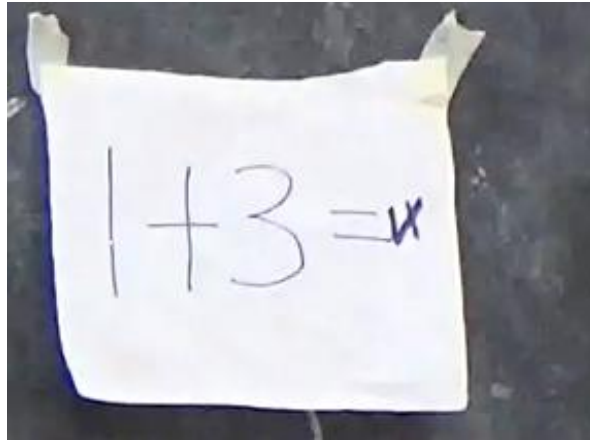


Figure 4-42: A solution by one group (Source: Researcher).

The teacher isolated the error using her technique of verbalising the hand movement that linked her talk and gesture to the chalkboard inscriptions. For the 4 shown in Figure 4-42, the hand movement given by the teacher was “Dot! Down! Go -up! Cut-in-the-middle!” In Figure 4-43 the teacher wrote the outcome of the hand movement: “Dot! Down! Go-up! Cut-in-the-middle!” that she said might have resulted in the 4 seen in Figure 4-42. The teacher highlighted the main difference being the angular turn in the acceptable movement: “Dot! Down! Turn-right! Cut-in-the-middle!”.

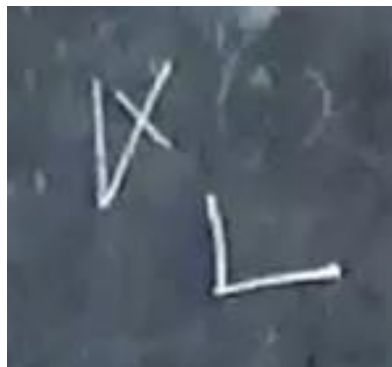


Figure 4-43: Teacher demonstrating the outcome of an incorrect hand movement for 4 (Source: Researcher).

The teacher explained in Utterance 216 of Excerpt 4-8, that the learners had earlier been taught these hand movements, such as the one for writing 4, when they were learning how to write

numbers. The teacher also used the same approach in the outset of Lesson 6, when a learner wrote a + sign that was rejected by the class (see Figure 4-44). To convince the one who wrote the sign why it was not accepted by the class, the teacher reminded the learner the original hand movement for the + sign discussed during Lesson 1: “Dot! Down! Cut-in-the-middle!” The teacher asked another learner to re-write the + sign by ensuring that the downward line is cut in the middle. A second learner was asked to rewrite the correct sign for emphasis (see Figure 4-44).

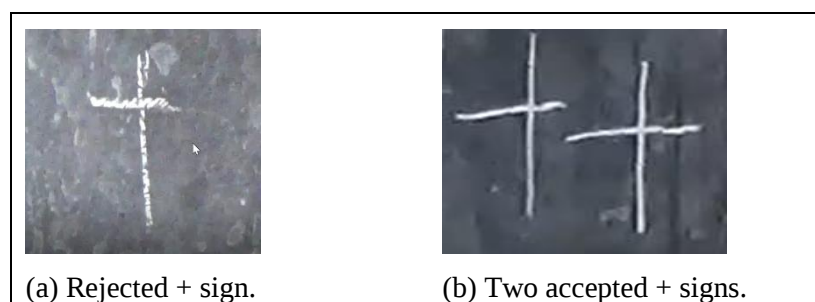


Figure 4-44: Rejected + sign and two acceptable signs (Source: Researcher).

4.3.6 Summary of the Standard 1 teacher’s use of mediational means

The Standard 1 teacher exemplified how to approach the concept of addition for the first time to learners. The teacher worked out the tasks and examples by linking her use of artefacts to their corresponding inscriptions, talk and gesture. The findings from the Standard 1 teacher demonstrated the connections that are made possible when the teacher switches between various means of mediation within the same task. The teacher made strong connections beyond the guidelines shown in the curriculum materials.

4.4 Use of mediational means in Standard 2

As indicated in Chapter 3, the three Standard 2 lessons were not observed consecutively. Two lessons were observed during the sixth week of the first term while the third lesson was observed during the 11th week. The teacher administered an assessment at the end of the sixth

week that included the content taught during the first two lessons. The insights from the assessment have been discussed in section 4.4.6.

4.4.1 An overview of Standard 2 lessons

Each of the Standard 2 lessons was segmented into five episodes.

Lesson 1

The teacher started the first episode of Lesson 1 by vertically presenting $12+5$ on the chalkboard and worked out the answer with the class. During Episode 2, the teacher worked out $1 + 9$ and $6 + 4$ with the class on the chalkboard. In Episode 3, the teacher asked two learners to solve $2 + 8$ and $5 + 5$ side-by-side on the chalkboard and verified the solutions with the whole class. In Episode 4, the teacher gave sheets of paper with three problems ($8 + 2$, $3 + 7$, and $5 + 5$) to groups and verified the solutions with the whole class. During Episode 5, the learners were then asked to solve $11 + 6$, $14 + 5$, and $15 + 3$ in their notebooks and marked by the teacher. The solutions were verified by the teacher and the whole class using counters. This lesson has been presented in a single-page lesson graph in Appendix 8.

Lesson 2

Episode 1 was a review of the previous lesson. In Episode 2, the teacher worked out $3 + 9$ with the class. During the third episode, the teacher asked two learners to work out $4 + 8$ and $5 + 7$ side by side on the chalkboard and verified the solutions with the whole class. In Episode 4, learners were given papers with $9 + 3$ and $8 + 4$ to solve in their groups and the solutions were verified by the whole class. In the last episode, learners were given $6 + 6$ and $7 + 5$ to solve in their notebooks, and thereafter verified by the whole class. The progression of Lesson 2 has been presented in the lesson graph shown in Appendix 9.

Lesson 3

During the first episode of Lesson 3, the teacher reviewed the previous work on counting up to 50 in Episode 1. In Episode 2, the teacher worked out $35 + 13$ with the whole class using two place-value boxes. In Episode 3, the teacher invited one learner to work out $28 + 11$ using place-value boxes and verified the solution with the whole class. Episode 4 focused on working out $45 + 2$ using place-value boxes by the whole class. During the last episode, learners were given $6 + 22$ and $36 + 10$ solved in their notebooks and marked by the teacher. The lesson graph in Appendix 10 shows how the lesson progressed.

4.4.2 Mediating tasks and examples

In Standard 2, learners were expected to work with numbers not exceeding 99 by the end of the school year.

Nature of tasks and examples observed in Standard 2

All the tasks involved finding the sum of two numbers, except for the first task of Lesson 3 in which learners were asked to count from 1 to 50. All the tasks during the first two lessons involved adding two numbers with a sum not exceeding 20. Tasks done during the third lesson had two addends with a sum not exceeding 50.

Types of tasks

During the three lessons, tasks were presented verbally, on the chalkboard, or on pieces of paper distributed to the learners in groups. Some tasks were executed as whole-class teacher-directed activities during which the teacher and the class worked together in finding the required solutions. Other tasks were independently done by learners who were invited to the front of the classroom to work out the required solution and thereafter verified by the teacher together with the class. For group tasks, learners were given problems to be solved independently. Thereafter, the problems were re-worked by the teacher and the whole class to

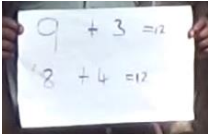
verify the answers found by the groups. The last task was mostly an individual exercise that was marked by the teacher and thereafter solved by the whole class to enable learners to verify correct solutions and correct errors.

Table 4-14: Types of tasks across the three Standard 2 lessons

Episode	Lesson 1	Lesson 2	Lesson 3
1	Whole class	Whole class	Whole class
2	Whole class	Whole class	Whole class
3	Two learners in front followed by the whole class	Two learners in front followed by the whole class	One learner in front followed by the whole class
4	Group work followed by the whole class	Group work	Whole class
5	Individual work followed by the whole class	Individual work followed by the whole class	Individual work followed by the whole class

The activities in similar episodes across the three Standard 2 lessons were the same. Some tasks were also done in the same manner as in Standard 1. For instance, Table 4-15 shows a summary of the activities in Episode 4 of Standard 2 Lesson 2 (also indicated in Table 4-16). An examination of the activities in Table 4-15 shows that the presentation of tasks in Episode 4 was done in the same way as the Standard 1 episodes shown from Table 4-6 to Table 4-8.

Table 4-15: Summary of Episode 4 of Standard 2 Lesson 2

4	4.1	Working out $9 + 3$ and $8 + 4$ (Groupwork)	The teacher distributes single sheets of paper to 8 groups of learners, each with all the addition statements $9 + 3 =$ and $8 + 4 =$. Asks for group representatives to line up in the front of the classroom displaying their written solutions on the given papers. 
	4.2	Verifying the solutions for $9 + 3$ and $8 + 4$ (Teacher and whole class)	Teacher and whole class use counters to verify the answers for $9 + 3 =$ and $8 + 4 =$ worked out in the preceding group work activity (still displayed by 9 representatives in front); which were all showing answers of 12. 

The teacher confirmed during an interview that the structure of the observed lessons was typical of her lessons in Standard 2. The teacher explained that her lessons start with an introduction. Thereafter, she tells the class that “today we will learn mathematics”. This would be followed by an example, then group work, as well as a discussion of the group work. She then gives individual work and marks the learners’ notebooks. The lessons end with revisions of the individual work.

Duration of tasks

The first two lessons were relatively shorter (see Figure 4-45). The third lesson, involving addition with place-value boxes, was completed in 1-hour 3 minutes.



Figure 4-45: The time spent on various types of tasks across the three Standard 2 lessons
(Source: Researcher).

While the time spent on individual work remained relatively the same during the first two lessons, the teacher spent twice as much time on whole-class tasks in Lesson 3. This was the case even though the whole-class tasks in Lessons 1 and 2 had two examples at a time while Lesson 3 had single examples (see Table 4-17). Despite handling fewer examples, Lesson 3 took 30 percent longer than Lessons 1 and 2. The implications of these remarkable durations have been discussed in section 5.6.2.

Considering the average time spent on similar types of tasks during each lesson, half of the time was spent on whole-class activities. This can be seen in Figure 4-46 that follows.

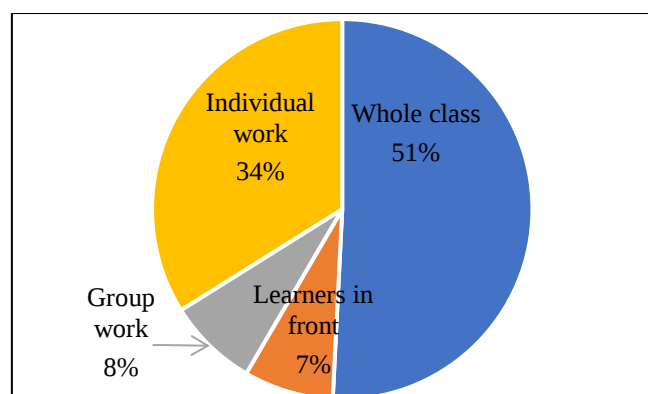


Figure 4-46: Average duration of task types in Standard 2 (Source: Researcher).

As shown in Figure 4-46, the individually done tasks also took a considerable portion of the lesson time.

Lesson sequence

The tasks in each lesson were presented systematically in 5 episodes. To illustrate how this was done, Table 4-16 shows how tasks were sequenced during the second lesson.

Table 4-16: Sequencing of tasks during Lesson 2

	Episode 1	Episode 2	Episode 3	Episode 4	Episode 5
Task	Review of the previous lesson	Finding the sum of two numbers	Finding the sum of two numbers	Finding the sum of two numbers	Finding the sum of two numbers
Presentation of task	Verbal	Chalkboard	Chalkboard	Prewritten papers	Chalkboard
Type of task	Whole class	Whole class	Two learners in front followed by whole-class verification	Group work followed by whole-class verification	Individual work followed by whole-class verification

In each of the three lessons, the first two tasks were done by the teacher and the whole class, while the third task was done by learners on the chalkboard. Except for Lesson 3, the fourth task was group work followed by whole-class verification. The last task in all the lessons was individual work marked by the teacher followed by corrections done by the teacher and the

whole class. This task pattern has been presented in Table 4-14. This pattern was also described by the teacher during an interview.

Examples used in Standard 2

The examples used by the teacher during the three lessons have been presented in Table 4-17.

Table 4-17: List of examples used in Standard 2

	Lesson 1	Lesson 2	Lesson 3
Episode 1	$12 + 5$	$5 + 5, 9 + 0, 8 + 2$	Counting up to 50
Episode 2	$1 + 9, 6 + 4$	$3 + 9$	$35 + 13$
Episode 3	$2 + 8, 5 + 5$	$4 + 8, 5 + 7$	$28 + 11$
Episode 4	$8 + 2, 3 + 7, 5 + 5$	$9 + 3, 8 + 4$	$45 + 2$
Episode 5	$11 + 6, 14 + 5, 15 + 3$	$6 + 6, 7 + 5$	$6 + 22, 36 + 10$

In Table 4-17, it can be seen that during Lesson 1 all the examples in Episodes 2 to 4 focused on number bonds or “pair-wise configurations” (Wright & Ellemor-Collins, 2018, p. 20) of 10; while during Lesson 2, all examples in Episodes 2 to 5 were on number bonds of 12. Most of the solutions to the presented problems were given by learners.

The opportunities of learning that were offered by the teacher’s use of the listed examples have been discussed under the teacher’s “mediating talk and gesture for building mathematical connections” in section 4.4.5. The rationale for the teacher’s selection of tasks and examples has been discussed in the next sub-section.

The rationale for the Standard 2 teacher’s choice of tasks and examples

The teacher’s selection of tasks and examples appeared to be based on the suggestions from the teachers’ guide and was also influenced by the availability of teaching and learning resources.

The rationale for the selection of tasks

As shown in Table 4-14, among the three Standard 2 lessons observed, Lesson 3 was the only one in which group work was not done. It was noted, however, that the teachers' guide suggested that modelling the addition of numbers using place-value boxes could be set as a group task (see instructions 1 and 2 shown in Figure 4-47). However, organising the task to be done in groups would require the teacher to prepare a large number of place-value boxes and sticks. Instead, the teacher opted to do the same task using whole-class activities during which learners were invited to the chalkboard to perform various sub-tasks using the two place-value boxes prepared in advance before the lesson. By the end of the lesson, the teacher had asked 42 individual learners to make various contributions to the class out of the 94 learners that were present on that day.

Regarding the average number of problems given to the learners as individual work, the teacher said during an interview that she mostly gave them two or three problems. During the interview, the teacher said that she had an enrolment of 113 children in her class. As such, she had to be cautious when giving individual work so that she could manage to mark. While pointing to the learners' mathematics textbook page with 16 problems given as individual work, the teacher ended utterance 404 of Excerpt 4-13 with: "...they really gave us many". Referring to this case, the teacher said that instead of leaving all the 16 problems for the learners to do as individual work, she apportioned some of the tasks to classwork as well as group work.

The rationale for the teacher's selection of examples

The teacher selected the examples from both the teachers' guide and the learners' textbook. Table 4-17 shows that the first three examples used in Lesson 3 were presented in the same sequence as suggested in the teacher's guide (see Figure 4-47).

Instructions

- 1 Ensure that each group of learners has sticks and a place value box.
- 2 Distribute cards with the following incomplete addition sentences to groups of the learners:

$$\begin{array}{r} 35 \\ + 13 \\ \hline \end{array}$$

$$\begin{array}{r} 28 \\ + 11 \\ \hline \end{array}$$

$$\begin{array}{r} 45 \\ + 2 \\ \hline \end{array}$$

- 3 Let the learners use place value boxes and sticks to complete the addition sentences.
- 4 Discuss with the learners how to complete the addition sentences.
- 5 Let the learners practise modelling addition using place value boxes.
- 6 Ensure that each group of learners has counters.
- 7 Distribute charts with the following incomplete addition sentences to groups of the learners:

	T	O
	4	2
+		7

	T	O
	1	6
+	3	3

	T	O
		9
+	4	0

- 8 Let the learners complete the addition sentences using counters.

Figure 4-47: Snippet from the Teachers' Guide for Standard 2 (Source: Malawi Institute of Education, 2012c, p. 28).

Even though the teacher's guide was the main source of classwork and the learners' textbook was for individual exercises, the teacher explained during the interview that she sometimes used both books for either purpose. To illustrate how she worked with the two sources, the teacher referred to a particular section in the teacher's guide (see Appendix 11) where she based the first two lessons. She then pointed to the corresponding work in the learners' textbook (see Appendix 12) and said the utterances in Excerpt 4-13 that follows:

404. T: So, there were basic addition facts of ten and twelve. So, when we go to the learners' book, those ones are not there. But the activity is telling us to do what? To do those. So, if you see here [*pointing to a page in the learners' textbook*], you can see that there are many, not so? A good number, they really gave us many.
405. R: Yes.
406. T: So, it was like I put the learners' book aside ... And picked some problems here [*pointing to a page in the teacher's guide*] as examples, and also gave them some from here as what? An exercise.
407. R: Ah? Okay.

Excerpt 4-13: Interview response to the teacher's selection of tasks and examples.

The teacher explained in Utterance 404 of Excerpt 4-13 that she sometimes felt a mismatch between the activities given in the teachers' guide and the corresponding tasks in the learners' textbook. During Lesson 1, the teacher focused on number bonds of 10 from the teachers' guide but she felt that the tasks given in the learners' textbook did not directly match number bonds. As such, she picked class tasks from both the teachers' guide as well as the learners' textbook and used them interchangeably as individual work or whole class work based on the situation.

4.4.3 Mediating artefacts

The Standard 2 teacher worked with three major types of artefacts.

Nature of artefacts

The teacher mainly worked with counters fitted to a frame, prewritten papers, and place-value boxes.

Framed counters

In all the three lessons, the teacher worked with framed counters (see Figure 4-2). During Lesson 3, the teacher also used counters for verifying the solutions to problems that had already been solved with place-value boxes. The rationale for the teacher's use of counters in parallel with place-value boxes has been discussed towards the end of this section (4.4.3).

Prewritten papers

In Episode 4 of Lesson 1, learners were given prewritten papers, each carrying three addition statements ($8 + 2$, $3 + 7$, and $5 + 5$) to be solved in groups (see Figure 4-48). During Episode 4 of Lesson 2, the teacher also used prewritten papers that carried two problems with a sum of 12.

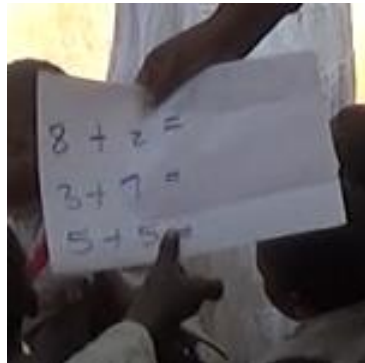


Figure 4-48: Prewritten sheets of paper presented to groups during Lesson 1 (Source: Researcher).

After working out the solutions of the problems on the prewritten papers during the first two lessons, the teacher asked representatives of each group in the classroom to line up in front to display their answers, as shown in Figure 4-58.

Place-value boxes

During Lesson 3, the teacher worked with two place-value boxes and counters. When using counters, learners who did not bring their counters to class were asked to use their fingers. A place-value box (see Figure 4-49) had three related physical representations: A bundle of ten sticks representing a ten; a single stick representing a one; and the whole place-value box representing a number.



Figure 4-49: A drawing of a place-value box showing 35 in the learners' mathematics textbook (Source: Malawi Institute of Education, 2012a, p. 31).

The drawing of the place-value box shown in the learners' textbook had separate compartments for tens and ones. The place-value box made and used by the teacher is shown in Figure 4-50.



Figure 4-50: A place-value box made by the teacher, holding 35 (Source: Researcher).

The teacher introduced the addition of numbers using place-value boxes during Episode 2 of Lesson 3 (see Figure 4-51). The place-value box was already introduced to the learners before this lesson when counting, modelling, and writing of numbers from 20 to 50. So, the learners had learnt how to represent a number using a place-value box, but they had not yet been taught how to add two numbers represented by two place-value boxes. As shown in Figure 4-51, the teacher placed the place-value boxes right below the numbers they represented on the

chalkboard. This one-to-one correspondence made the representations of the numbers more noticeable to the learners.

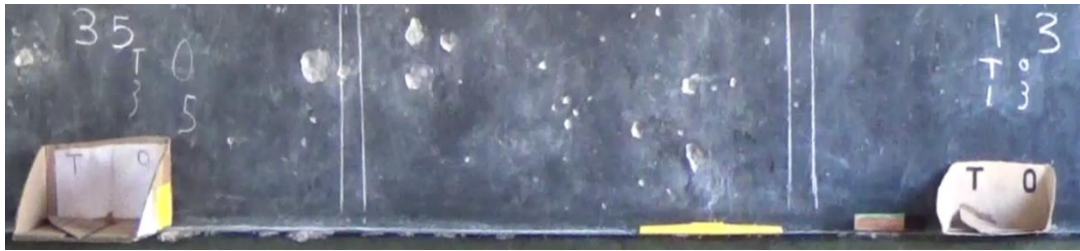


Figure 4-51: Placement of place-value boxes beneath numbers to be represented (Source: Researcher).

The teacher explained the process of addition using the place-value boxes focusing on the placement of the bundles and single sticks representing tens and ones respectively.

Fingers

Learners were also asked to use fingers during the lesson if they did not have counters (see Utterance 281 from Excerpt 4-14).



Figure 4-52: Learners without counters using fingers during Lesson 3 (Source: Researcher).

The Standard 2 teacher's use of artefacts

The artefacts were mainly used in the process of calculating the sum of the given numbers. In all the three lessons, the teacher's use of artefacts involved unit counting.

Use of counters

Counters remained the most versatile artefact used. As stated in the outset of the section (4.4.3), counters were even used to check the correctness of solutions already found using place-value boxes in Lesson 3. As indicated in Utterances 279 and 281 in Excerpt 4-14 from Lesson 1, the teacher checked and ensured that each learner was counting during the lesson, thus promoting learner engagement.

- | | |
|-----------|--|
| 277. T: | He says fourteen plus five equals. He is correct, alright? |
| 278. C: | Yes! |
| 279. T: | Let us count fourteen. Everyone should be counting! |
| 280. C+T: | <i>[Teacher pushes pieces of counters]</i> One, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve, thirteen, fourteen! |
| 281. T: | Those of you who are lying, sit down and count properly. Have you counted your fingers and toes and exhausted them all? |
| 282. C: | Yes! |
| 283. T: | Do you have some remaining? |
| 284. C: | Yes! |
| 285. T: | Ooh! Let us continue with five! |
| 286. C+T: | <i>[Teacher pushes pieces of counters]</i> One, two, three, four, five! |
| 287. T: | Let us count all of them together! |
| 288. C+T: | <i>[Teacher pushes pieces of counters]</i> One, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, nineteen! |
| 289. T: | Who can write nineteen for us? |

Excerpt 4-14: Finding $14 + 5$ using counters.

In Utterance 281, the learners who had not brought their counters were asked to use their fingers and toes.

Use of prewritten papers

Regarding prewritten papers shown in Figure 4-48, the teacher used them in a structured way during Episode 4 of Lessons 1 and 2 to show relationships that lead to number bonds of 10 and 12. The papers were all displayed at the same time by learners standing in the front of the classroom (see Figure 4-58). The way the teacher used these papers for showing connections

will be discussed when looking at the teacher's "mediating talk and gesture for building mathematical connections" in section 4.4.5.

Use of place-value boxes

After discussing with the class on the use of bundles and single sticks when adding numbers using place-value boxes, the teacher asked learners to do the same in subsequent examples. The teacher gave more than enough bundles and single sticks to a learner who had offered to compose the number 13 using a place-value box even though only one bundle and three sticks were required.

When finding the sum, the teacher and the class counted all the bundles and single sticks. This can be seen in Excerpt 4-15 when the teacher was determining the answer for $36 + 10$:

618. T: ... Now let us count how many bundles are there and how many ones are there. Let us count the bundles!
619. C+T: *[Count as the teacher places the bundles]* One, two, three, four!
620. T: Let us count the ones!
621. C+T: *[Count as the teacher places the sticks]* One, two, three, four, five, six!
622. T: Who can tell us what the number is? What is this number that we have formed?

Excerpt 4-15: Counting-all bundles and single sticks when adding 36 and 10.

The counting of all the single sticks and bundles to find the sum has been discussed under the teacher's "mediating talk and gesture for providing methods for generating solutions" under section 4.4.5. The bundles were also counted as "One, two, three, ..." to be discussed under teacher's "mediating talk and gesture for advancing learning connections" under the same section 4.4.5.

The rationale for the Standard 2 teacher's use of artefacts

The teacher's use of artefacts was partly suggested by the teachers' guide. The instructions numbered 6, 7, and 8 from the teachers' guide shown in Figure 4-47 presented the use of

counters independently from the use of place-value boxes. However, during Lesson 3, the teacher opted to use counters in parallel with place-value boxes. The teacher used the counters for verifying the solutions that were found with place-value boxes.

As shown in utterance 281 of Excerpt 4-14, the use of counters also acted as a way of promoting active learner engagement in her large class which had 86 learners present during Lesson 1. It seems the use of counters made it easier for the teacher to identify non-participating learners during the lesson. In four instances during Lesson 2, the teacher was able to note those who were not counting.

This was also emphasized by the teacher in utterances 176 and 180 in Excerpt 4-16 that follows:

176. T: Yes, we teachers see that children understand more when they do things by themselves. 178. R: Ehe? 179. T: Unlike when you do for them. 179. R: Ehe? 180. T: So, during a lesson, we try to involve the children, ... during the lesson, alright? 181. R: Yes.

Excerpt 4-16: The Standard 2 teacher's explanation of the importance of learner participation.

Utterances 176 and 180 in Excerpt 4-16 show that the Standard 2 teacher believed that learners understand more when they are involved in doing the tasks, which is achieved through the use of artefacts. During Episode 3 of Lesson 1, two learners went to the front to find the sum of two pairs of numbers, but they did not carry counters with them. It could be assumed that they were ready to use their techniques (such as their fingers), but the teacher insisted that their fellow learners should lend them counters before starting to work on the problems. During a follow-up interview, the teacher said that learners assume the role of a teacher when they go to the front. As such, those learners are always expected to use counters because they are more visible to the rest of the class.

4.4.4 Mediating inscriptions

The teacher worked with chalkboard inscriptions when presenting examples to be worked on during the lessons.

Nature of inscriptions in Standard 2

During the first two lessons, the teacher presented all the examples using structured mathematical statements. The teacher presented them on the chalkboard as incomplete statements and the answers were written by learners (see Figure 4-53).

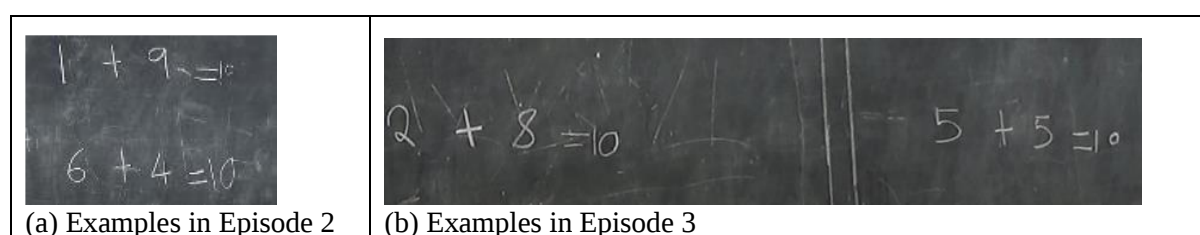


Figure 4-53: Presentation of examples in pairs during Lesson 1 (Source: Researcher).

The Standard 2 teacher's use of inscriptions

As shown in Figure 4-53, the examples were presented in pairs on the chalkboard during Lessons 1 and 2. Across the three lessons, the teacher started by asking learners to read the written statements. For instance, after writing 35 and 13 on the chalkboard during Lesson 3 (see Figure 4-54), the teacher invited the learners to read the two numbers. Thereafter, the teacher asked the learners to re-write the numbers using place-value headings T and O.

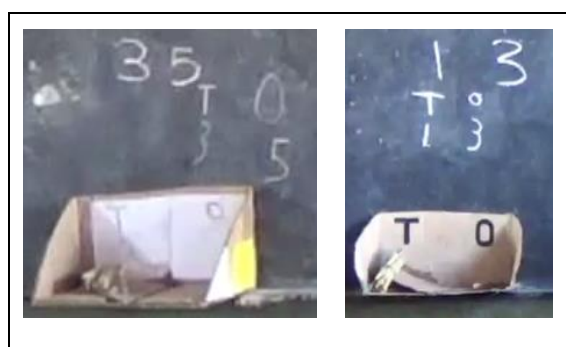


Figure 4-54: Inscriptions for working out $35 + 13$ by the teacher and learners (Source: Researcher).

After finishing solving the problem using the place-value boxes, the teacher wrote the problems again as structured mathematical statements (see Figure 4-55).

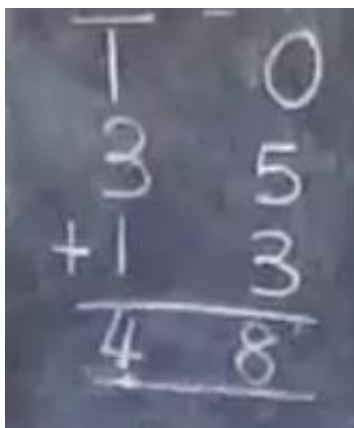

$$\begin{array}{r} 10 \\ 35 \\ + 13 \\ \hline 48 \end{array}$$

Figure 4-55: A structured mathematical statement for $35 + 13$ by the teacher (Source: Researcher).

The rationale for the Standard 2 teacher's use of inscriptions

The teacher mostly presented the inscriptions in such a way that learners should also have the opportunity to write on the chalkboard. This enabled them to practice the writing of new numbers that they had learnt during the preceding weeks. Although not all learners had the opportunity to go and write on the chalkboard and the sheets of paper given during group work, everyone had the opportunity to write during individual work.

4.4.5 Mediating talk and gesture

Mediating talk and gesture for providing methods for generating solutions

The introductory example ($12 + 5$) in Lesson 1 was the only one that was written vertically, and the teacher used column addition to add the ones and the tens separately. The teacher's talk with the learners indicated that they had already been familiarised with "where to start from" when given such tasks. She was adding ones separately even though she decided not to

explicitly mention it. The teacher used unit counting to find the sum of 2 and 5. After finding $2 + 5$, the teacher's talk proceeded with the addition of tens as shown in Excerpt 4-17:

33. T: *[Pointing to 1 on the chalkboard, almost touching the number]* What number are we remaining with here?
34. C: One!
35. T: One plus what? *[Points to the space to the right of the equal sign, above the first horizontal bar]*
36. C: Zero!
37. T: Zero! One plus zero, what is the answer? Who can tell us?
38. C: One!
39. T: One plus zero?
40. C: Madam! Madam!...
41. T: *[Points to Learner 4]* Eehh?
42. L4: One!
43. T: One! Thank you very much. He is correct, alright?
44. C: Yes!

Excerpt 4-17: Discussing the presence of 0 in $12 + 5$.

The responses from the class in Excerpt 4-17 show that the learners had been taught previously that there is an invisible zero which acts as a placeholder on the space between the + sign and the digit that comes after it, as shown in Figure 4-56(b). At this stage, the curriculum expectation was that learners should add two numbers with a sum less than 20. This means that they may only have encountered cases where both addends had single digits, or one addend had a single digit with the invisible 0 preceding it.

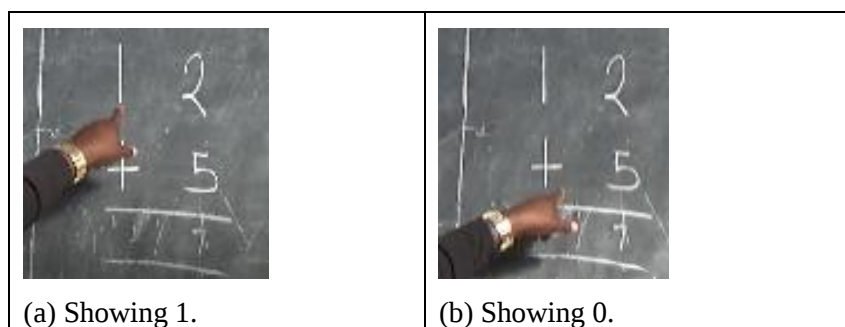


Figure 4-56: Teacher's gesture showing 1 and 0 (Source: Researcher).

The task of finding $12 + 5$ in Episode 1 of Lesson 1 was done in a relatively short time using the column addition strategy.

Counting all

Throughout the three lessons, the teacher used the count-all strategy to find and verify solutions to all the examples using counters. During a follow-up interview, the teacher indicated that counting-on would work well for fast learners in the classroom who would easily grasp that two quantities have been added together without necessarily experiencing those two separate quantities. A case in point was the example $14 + 5$ in Episode 5, which had the largest sum among all the 11 examples of Lesson 1. After counting up to 14 during the lesson, the teacher said: “Ooh! Let us continue with 5!”, giving some indication that she would count on. However, she proceeded to count 5 and followed by counting-all to find the answer. During a follow-up interview (see Excerpt 4-18 that follows) the teacher recounted how $14 + 5$ could be found without necessarily counting up to 19, but by using place-value addition:

251. T: So, for example, when finding $14 + 5$, ...then 4 should be added with 5, which are all ones. Alright?
252. R: Yes
253. T: This means that five, ... 1 will be added with zero.
254. R: Yes.
255. T: So, when adding, the children, we tell the children that: "When adding we start from where? From ones".
256. R: Yes.
257. T: So, we say: "Let us count four!", We start counting four: " One, two, three, four " "We should add what?" " Five!" "Let us count five!" "One, two, three, four, five!" "Let us count all of them together!" Then we start again: " One, two, ..." to see that if we add four and five together, how much do we get? So, we start counting: "One, two, three, four ..." up to nine.
258. R: Mmmm.
259. T: Then we write nine. Alright?
260. R: Mmm (Yes).
261. T: Then we wri...say: " One! plus...what number is here?" The children will then see that there is nothing. "If there's nothing, it means there is what?" "There is 0."... "So, one plus zero (which means nothing) is what?" Then the children will say: "One!" But to some people ... to some children, for them to understand you tell them: "Let us count one!", " One!" you then do the counting with the counters. "We should add is zero!", "Zero!" "This means we will not do what?" "We will not add ". "Let us count how many we have?", "One!"
262. R: Mmm (Yes).
263. T: Aha. "So, what do we write? ... The one we had at first."
264. R: Mmmh.
265. T: Yes. But counting all of them up to nineteen? ...Aaah. No.

Excerpt 4-18: Interview with the teacher on finding $14 + 5$.

In Excerpt 4-18 the teacher explains how she would work with her learners to find $14 + 5$ without necessarily doing unit counting up to 19. She indicated in Utterance 265 that unit counting up to 19 when solving $14 + 5$ would not be ideal. Instead, the class would be asked to start with adding ones (Utterances 251 and 255) followed by " $1 + 0$ " (Utterance 261). This explanation is the same strategy that the teacher used in finding $12 + 5$ at the beginning of Lesson 1. The strategy seems to be a localised method for example-sets whose sum was less than 20, and one of the two addends had two digits.

As shown in Excerpt 4-19, the learners appeared to have some potential to do more than what they were formally expected to do. In Utterance 38 of Excerpt 4-19, the learner who raised a hand answered “12” at a time when the teacher was expecting the learners just to read the given statement ($3 + 9$).

35. T: *[Picks the Teachers' Guide and writes $3 + 9 =$ on the chalkboard]* Can everyone look at the chalkboard! I have written some number on the chalkboard. Who can read for us?
36. C: *[Raising hands]* Madam! Madam!...
37. T: ... *[Points to Learner 7]* Eeh!
38. L7: Twelve.
39. T: *[Pointing to $3 + 9 =$ on the chalkboard]* Have I written twelve here?
40. C: No!
41. T: Who can tell me this number?
42. C: *[Raising hands]* Madam! Madam!...
43. T: *[Points to Learner 8]* Mhm!
44. L8: Three plus nine equals.
45. T: He says three plus nine equals. He is correct, alright?
46. C: Yes!

Excerpt 4-19: A learner giving the solution before the formal addition procedure.

After the discussion in Excerpt 4-19, the teacher continued with the formal procedure for adding the two numbers using counters, as shown in Excerpt 4-20:

49. T: Three plus nine equals! Everyone should pick their counters. Three plus nine! Let us three together! I want to see everyone counting, alright?
50. C: Yes!
51. T: Let us count three together!
52. C+T: *[Teacher pushes counters]* One, two, three!
53. T: Let us add nine!
54. C+T: *[Teacher pushes counters]* One, two, three, four, five, six, seven, eight, nine!
55. T: Let us count all these to see how many they are altogether!
56. C+T: *[Teacher pushes counters]* One, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve!
57. T: This means that three plus nine gives twelve as the answer, alright?
58. C: Yes!

Excerpt 4-20: Formal count-all procedure when finding $3 + 9$.

As seen in Excerpt 4-20, the answer was formally found in Utterance 57, yet Learner 8 had found it earlier in Utterance 38 within Excerpt 4-19.

Representation of addition using place-value boxes during lesson 3

At the beginning of Lesson 3, the teacher reinforced the meaning of bundles of sticks and single sticks when representing numbers in a place-value box. For all the examples discussed during the lesson, the teacher started by discussing with the class the representation of each addend in a place-value box. The addition process was expressed as putting together tens with other tens and ones with other ones. The teacher practically demonstrated this process of addition in the first example by adding the contents of the box for 13 to the box for 35 (see Figure 4-54).

The learners seemed to find it easier and quicker just to work out the sum of two numbers by just writing than by representing the numbers physically. During the lesson, one learner had already arranged 28 and 11 in two place-value boxes, but when the teacher asked him to go ahead and add the represented numbers, he went where the teacher had already written 28 on the chalkboard and appended + 11 followed by a horizontal bar at the bottom, as shown in Figure 4-57. The teacher asked him to stop writing and redirected him to the place-value boxes.



Figure 4-57: Learner appending + 11 to 28 to solve on the chalkboard (Source: Researcher).

When adding numbers using place-value boxes, it was noted that the teacher started working with bundles followed by single sticks. This meant that the sum was found by first adding tens followed by ones. After finding the sum of two numbers using place-value boxes, the class was asked to “read” the number physically represented by the added bundles and single sticks. This was done by counting the tens and ones in the box carrying the sum.

Mediating talk and gesture for building mathematical connections

In almost all the examples done during the three lessons, the teacher only presented the example in writing and thereafter asked learners to take turns writing the answers on the chalkboard. During Lessons 1 and 2, the teacher’s talk made connections between examples that had been solved by learners. For instance, after two learners had solved $2 + 8$ and $5 + 5$ written side by side during Lesson 1 (see Figure 4-53), the teacher talked about the connection between the examples in Excerpt 4-21:

141. T:	Thank you very much. Go and sit down. This one has added two plus eight and has found that the answer is ten. That one too has added five added to five and has also found that the answer is what?
142. C+T:	Ten! <i>[Teacher also mentions ten]</i>

Excerpt 4-21: Teacher showing the similarity between $2 + 8$ and $5 + 5$.

After the learners solved the problems in groups the teacher also made a connection among the given examples as number bonds of 10. This was done visually by lining-up all the group representatives in front to show their group’s solutions as shown in Figure 4-58.

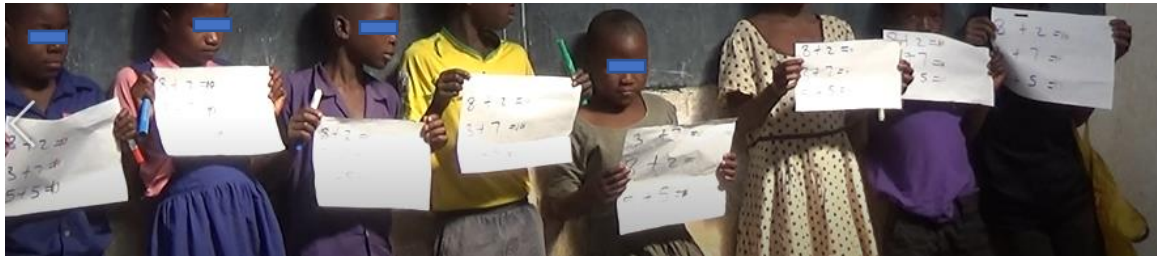


Figure 4-58: Showing similarity of three examples ($8 + 2$, $3 + 7$, and $5 + 5$) during Lesson 1
(Source: Researcher).

On the pieces of paper given to groups, the three problems were written in alignment, making it easy for the learners to see that the sum was 10. In her talk, the teacher made emphasis on the similarity among the examples solved by the groups in Excerpt 4-22 that follows:

- | | |
|-----------|--|
| 183. T: | Can we all see these groups? |
| 184. C: | Yes! |
| 185. T: | Are the answers similar? |
| 186. C: | Yes! |
| 187. T: | What answers have they all found? |
| 188. C: | Ten! |
| 189. T: | Ten! Alright? |
| 190. C: | Yes! |
| 191. T: | The first one, eight plus two. The one standing there has found ten, the other one ten, the other ten, the other ten, the other ten, the other ten, the other one also ten, alright? |
| 192. C: | Yes! |
| 193. T: | Problem number two; three plus seven, they have all also found ten. The last one, five plus five, they also have found that it is what? |
| 194. C+T: | Ten! |

Excerpt 4-22: Teacher emphasising similarity among three examples.

Throughout Lesson 3, the teacher linked artefacts and inscriptions with talk and gesture. The teacher's connections between inscriptions with talk and gesture has been shown in Figure 4-56 and Figure 4-5 in section 4.2.4. Her mediating talk can be seen in Excerpt 4-23 below when

the teacher was finalizing $45 + 2$ using column addition. This explanation was meant to verify the answer that was found earlier using place-value boxes. After adding the ones, she had to show learners that the corresponding digit under tens for 2 is 0.

- | | |
|-----------|---|
| 489. T: | ... If there is no number what is there? <i>[Moving the hand around the space below 4]</i>
There is zero. There is nothing, alright? |
| 490. C: | Yes. |
| 491. T: | There is nothing. So, we will just take four and put it here. Alternatively, when adding we will count four. Let us count four! |
| 492. C+T: | <i>[Teacher pushes pieces of counters as learners count]</i> One, two, three, four! |
| 493. T: | Let us add zero! For zero we will not put anything, alright? <i>[Gesturing with hands]</i> |

Excerpt 4-23: Teacher's use of gesture when referring to zero.

The teacher's gesture when referring to zero in Excerpt 4-23 has been shown in Figure 4-59.



Figure 4-59: Teacher gesturing zero with hands (Source: Researcher).

Use of language

Based on speech markers that she used, the teacher sometimes appeared to have used a singular reference of the Chichewa word *nambala* [number] even if she seemed to refer to two addends in an addition statement. Towards the end of the lesson, the teacher just referred to the addition statements as “first one” and “second one”.

Mediating talk and gesture for advancing learning connections

The Standard 2 teacher had some aspects that enabled the advancement of learning

connections.

Verification of offers

The teacher worked with several errors made by learners during the three lessons. For instance, during the review of previous learning in Lesson 2, a learner mentioned $9 + 0$ as an example that was learnt during the last lesson on number bonds of 10. Even though this example was not discussed in that lesson, and was not a number bond of 10, the teacher opted not to refute. The teacher did not always disregard incorrect offers from learners but addressed them appropriately. During the same Lesson 2, another error was made by a learner who was assigned to solve $4 + 8$ on the chalkboard. The learner worked out the answer with counters but when writing he took a long time thinking and hesitantly wrote 10. The teacher verified the offer using counters. In some cases, verification of learners' offers required several learners making attempts. For example, at one point during Lesson 3, the teacher asked learners to write 39 on the chalkboard. Three learners came forward in turns; the first learner wrote 29, the second learner wrote 59, and the third learner correctly wrote 39. During each turn, the teacher asked the class what number had been written before asking someone else to make another attempt.

During the three lessons, the teacher mostly repeated learners' offers followed by asking the question, "this is correct, alright?" There were some isolated instances where learners would answer "no!" to this question. The teacher also asked some questions that encouraged the learners to think, such as "what should we add?" or "what number are we remaining with here?"

Reinforcement

When learners had given the correct answer, the teacher gave them a positive reinforcement. In some cases, this required the learner to choose the type of handclap they would prefer as seen in Utterances 439 and 441 of Excerpt 4-24 that follows:

- 439.T: *[Talking to Learner 26]* Thank you very much. Go on sit down. *[Talking to Learner 27]* The one who composed forty-five, can you stand up! What type of handclap would you prefer?
440. L26: *[Stands up]* CRECCOM clap.
441. T: She says CRECCOM style. Do not be jealous! Give a nice CRECCOM hand for your friends, alright?
442. C: Yes!
443. T: Let us do the CRECCOM clap!
444. C: *[Clap hands using the CRECCOM style]*
- 445.T: Thank you very much.

Excerpt 4-24: Positive learner reinforcement during Lesson 3.

Repetition

The teacher reinforced the correct meanings and representations using repetition. The class was sometimes asked to repeat mentioning a number after it was correctly read by a fellow learner from the chalkboard. In Excerpt 4-25 that follows, the teacher reinforced the meaning of a bundle by repeating up to six times:

124. T: So, we said that when we group items like this...in a bundle like this, how many items are there? *[Holding some bundles of sticks in her hands]*.
125. C: 20!
126. T: How many items are there?
127. L11:*[One learner heard quickly interrupting the teacher shouting ten!]*.
128. C: Ten!
129. T: How many items do we have here? *[Holding one bundle up in the right hand]*.
130. C: Ten!
131. T: How many items are here? *[Holds up another bundle which was kept with other bundles in the left hand]*.
132. C: Ten!
133. T: How many items do we have here? *[Holds up a third bundle in the right hand]*
134. C: Ten!
135. T: What about here, how many items are also here? *[Holds up a fourth bundle in the right hand]*.
136. C: Ten!
137. T: Ten! Now, for tens *[Pointing at T in the first place-value box under 35]*, who can tell me how many bundles should we put under tens for the number thirty-five? *[Pointing at the number 35 written by the teacher on the chalkboard]*.

Excerpt 4-25: Reinforcing the meaning of a bundle through repetition.

In utterance 125, the learners might have rightly responded 20 because by the time the question was being asked, the teacher was not only holding one bundle. Also, the question was initially phrased as “...when we group items like this...” making some learners think beyond a bundle before the clarification by the teacher.

4.4.6 Insights from a weekly assessment

The scripts that were written by learners during the assessment done after Lesson 2 were examined to look for clues that could be related to the teachers’ explanations used during the lessons. One of the questions had $14 + 9$ that was solved during individual work in Lesson 1 (see Excerpt 4-14) and was also discussed by the teacher during an interview (see Excerpt 4-18). Figure 4-60 shows some of the learners’ work that gave correct solutions to the assessment items of addition of numbers up to 20.

Learner 1	Learner 2	Learner 3

Figure 4-60: Some of the learners’ work that gave correct solutions to assessment tasks (Source: Researcher).

Some of the learners made mistakes in their calculations as shown in Figure 4-61. The mistakes seemed to be due to counting errors.

Learner 4: $14 + 5 = 20$	Learner 5: $14 + 5 = 20$	Learner 6: $14 + 5 = 18$

Figure 4-61: Learners’ mistakes that could be attributed to counting errors (Source: Researcher).

The learners' work shown in Figure 4-61 missed the correct answer with a ± 1 error, while in Figure 4-62, the pattern of errors seems to indicate that some learners probably had a similar conceptual error.

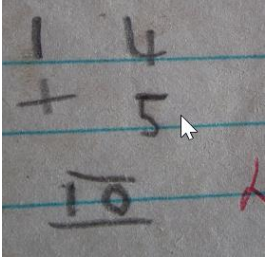
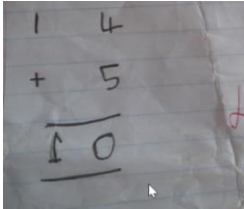
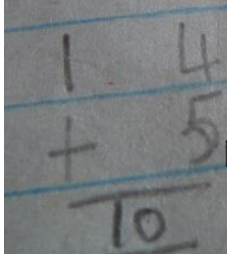
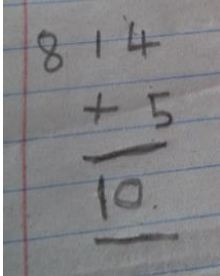
			
Learner 7: $14 + 5 = 10$	Learner 8: $14 + 5 = 10$	Learner 9: $14 + 5 = 10$	Learner 10: $14 + 5 = 10$

Figure 4-62: Learners' mistakes that could be attributed to conceptual errors (Source: Researcher).

The errors in Figure 4-63 seem to be related to the learners' ability to write the number correctly after working out the answer correctly.

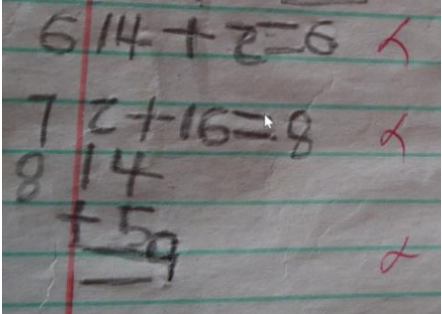
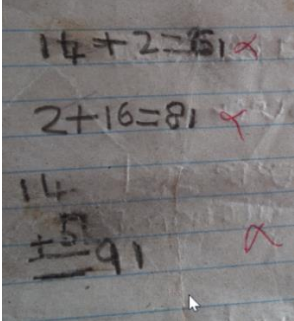
	
Learner 11: Writing ones digit only	Learner 12: Reversing the order of digits

Figure 4-63: Learners' mistakes that could be attributed to writing errors (Source: Researcher).

In Figure 4-63, Learner 11 consistently skipped the ten when writing 16, 18, and 19, presenting them as 6, 8, and 9, respectively. Learner 12, on the other hand, made a transposition error for all the three answers, writing 16, 18, and 19 as 61, 81, and 91, respectively.

4.4.7 Summary of the Standard 2 teacher's usage of mediational means

The Standard 2 teacher exemplified connections within and across examples. The teacher made strong connections between various means of mediation within an example during Lesson 3, while mathematical connections across examples were stronger during Lessons 1 and 2. The teacher also demonstrated the structured use of artefacts during Lesson 3 when working with place-value boxes. Structured use of inscriptions was noted in the way examples were presented on the chalkboard, such as presenting them in pairs during the first two lessons. The teacher's talk and gesture helped in making the mathematical connections within and across examples noticeable to the learners.

4.5 Use of mediational means in Standard 3

As pointed out in Chapter 3, the Standard 3 teacher was teaching mathematics for the first time since her completion of teacher training. Four lessons were observed and were taught successively during the eighth week of the first term.

4.5.1 An overview of Standard 3 lessons

The first lesson was segmented into three episodes while the last three lessons were each parsed into four episodes.

Lesson 1

The first episode was a review of skip counting in 10s from 300 to 600. During Episode 2, the teacher worked with the whole class in finding $442+105$ using abaci that were distributed to the learners in groups ranging from 10 to 15 learners per group. In the last episode, learners were asked to work out $574 + 2$, $207 + 112$, and $351 + 36$ individually in their notebooks and marked by the teacher. This was followed by verification of the solutions by learners who were invited to the front to solve the problems using counters. The progression of Lesson 1 has been presented in the lesson graph shown in Appendix 13.

Lesson 2

Lesson 2 started with a review of skip counting in 10s from 400 to 600 in Episode 1. In Episode 2, the teacher worked with the whole class in working out $541 + 27$ using abaci. In the third episode, learners were asked to work out $412 + 167$ in groups using abaci. During the last episode, learners were asked to work out $425 + 42$ and $361 + 128$ in their notebooks and marked by the teacher. The solutions were verified by two learners on the chalkboard. The lesson graph in Appendix 14 shows how Lesson 2 progressed.

Lesson 3

This lesson started with skip counting from 400 to 600 in the first episode. The second episode involved finding $346 + 138$ using abaci. In Episode 3, learners were asked to work out $263 + 129$ in their groups using abaci. The solutions were verified by a learner who worked out the same problem in front. In Episode 4, learners were given $318 + 242$ and $375 + 17$ to be solved in their notebooks and marked by the teacher. The solutions were verified by two learners on the chalkboard. The progression of Lesson 3 has been presented in the lesson graph in Appendix 15.

Lesson 4

The first episode started with skip counting in 10s from 500 to 600 by the whole class. Some individuals were then asked to skip-count the same range in 5s. The second episode involved working out $327 + 118$ by the teacher and the whole class. During Episode 3, learners were asked to solve $519 + 6$ in groups using abaci. Episode 4 had two problems ($376 + 19$ and $126 + 439$) that were solved by learners in their notebooks and marked by the teacher. The solution to the first problem was verified by a learner in front of the classroom while the second one was verified by the teacher and the whole class. The lesson graph for Lesson 4 has been presented in Appendix 16.

4.5.2 Mediating tasks and examples

By the end of Standard 3, learners are expected to be able to add two numbers with a sum not exceeding 999. It is at this level where addition involving regrouping is introduced.

Nature of tasks and examples observed in Standard 3

All the observed Standard 3 lessons started with an introductory episode where the teacher asked the class to do skip-counting of numbers between 300 and 600. The skip-counting range corresponded with the range of sums obtained from the rest of the tasks done during the lessons. The other tasks required finding the sum of two numbers not exceeding 600.

Types of tasks

Just like in Standards 1 and 2, the tasks in Standard 3 were done by the teacher and the whole class, through group work, by a learner in the front of the classroom, or as individual work. In some episodes, the teacher used more than one type of task. For example, in the last episode of Lesson 1, the teacher started by working with the whole class in finding the solution to one problem and asked two learners to take turns solving the last two problems. Some tasks were done with learners in groups, but the instructions at each stage seemed to be directed to the whole class. Such groups seemed to be formed for mainly sharing resources with no group-level discussion. This was more noticeable in Episode 2 of Lesson 2 where the learners were following instructions while sharing abaci in groups (formed by seating arrangement, learners seated close to each other shared the abaci), but during Episode 3 they were asked to go to their assigned groups. The teacher had earlier opened Episode 2 by saying: “Now, we will do an example!” to make the learners recognise that it was a whole-class task even if they had to do it with their neighbours.

Duration of tasks

The amount of time that the teacher spent on each type of task across the four lessons has been presented in Figure 4-64.

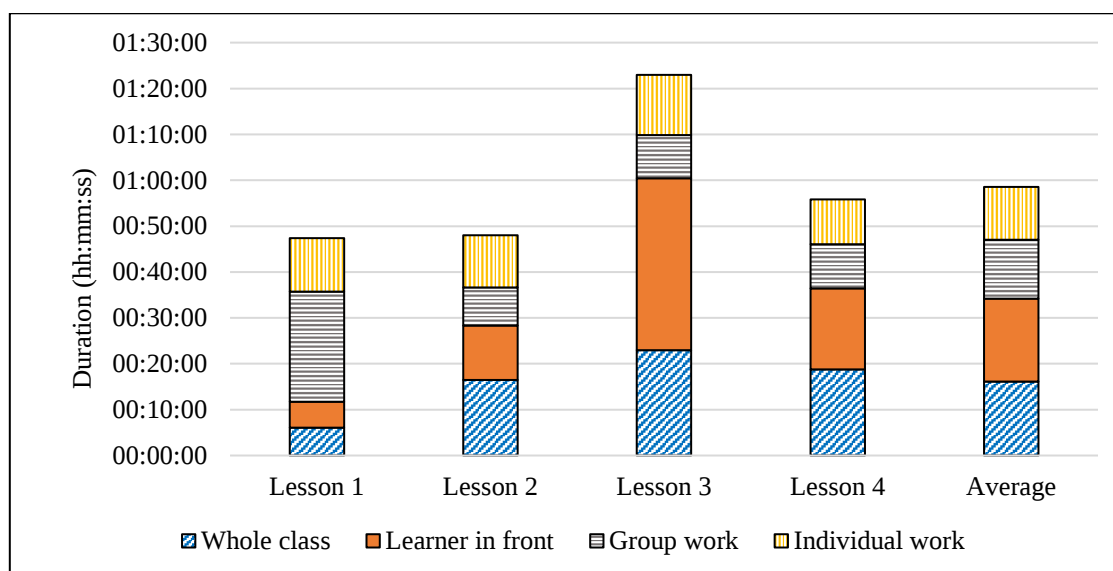


Figure 4-64: The time spent on various types of tasks across the four Standard 3 lessons (Source: Researcher).

It can be noted in Figure 4-64 that each lesson had its task type that dominated the lesson time. For example, there was much time spent on group work during the first lesson, but the time spent on group work remained fairly the same during the rest of the lessons (ranging from 8½ to 9½ minutes). During Lesson 2, much time was spent on whole-class activities while much of the time during Lesson 3 was spent with some learners in the front of the classroom verifying the solutions found from group work and individual work using pairs of abaci. It can also be seen in Figure 4-64 that the amount of time that was spent on individual work across all the four lessons was not much different, ranging between 10 and 13 minutes. It can also be noted in Figure 4-64 that all the tasks for a particular lesson were completed in more than the official time (period) allocated for a lesson (35 minutes). On average, the tasks done by learners turned

out to be the most prevalent type of interaction used by the teacher across all the lessons. This has been illustrated in Figure 4-65.

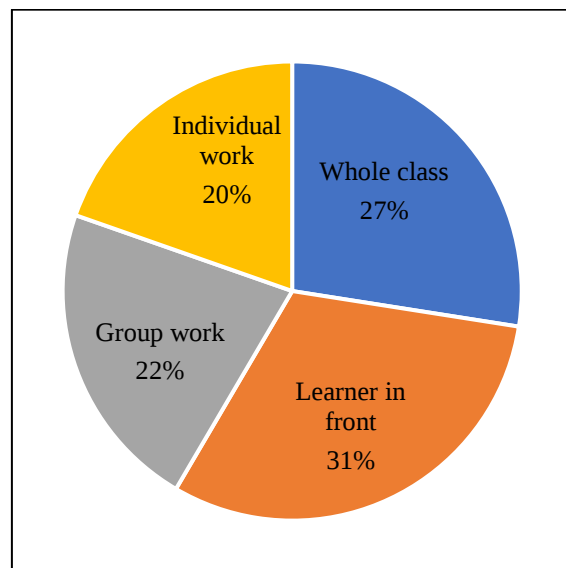


Figure 4-65: Average duration of task types in Standard 3 (Source: Researcher).

For all the four Standard 3 lessons, the teacher combined the two periods allocated for mathematics in a day, which is 70 minutes. The average overall lesson time for all the four lessons was 59 minutes as shown in Figure 4-64.

Lesson sequence

The sequencing of tasks in the Standard 3 lesson has been exemplified by the third lesson. The lesson was broken down into four major episodes corresponding to the tasks, as shown in Table 4-18.

Table 4-18: Tasks carried out during Standard 3 Lesson 3

Episode		Task	Nature of task
1		Review of skip counting from 300 to 600.	Teacher and whole class.
2		Finding $346 + 138$ using abaci.	Teacher and whole class.
3	3.1	Finding $263 + 129$ using abaci.	Group work.
	3.2	Verifying the solution for $263 + 129$.	A learner in front.
4	4.1	Finding $318 + 242$ and $375 + 17$.	Individual work.
	4.2	Verifying solutions for $318 + 242$ and $375 + 17$.	Two learners in front.

Just like in Standards 1 and 2, the solutions that were found through group work, or individual work were verified. During Lesson 3, the verification was done by three learners who solved the problem in front, with support from the class. In some cases, in the other lessons, the verification was done by the teacher. The class activities that took place during Episodes 2, 3, and 4 in Table 4-18 correspond to the chalkboard work illustrated in Figure 4-4.

The flow of lessons agreed with the explanation of the Standard 3 teacher during an interview. Her sequence was the same as those given by the teachers for Standards 1, 2, and 4. She said that after the introduction, she does some examples followed by group work, individual work, ending with an opportunity for learners to do corrections of the solutions found during individual work. The teacher also explained that the suggestions on how a particular lesson should flow are sometimes given in the teachers' guide.

Examples used in Standard 3

During the four lessons, the teacher worked with two 3-digit numbers with sums ranging between 300 and 600 as specified in the curriculum at this stage. As shown in Table 4-19, the lessons also started with the counting of numbers between 300 and 600.

Table 4-19: Skip counting ranges used during the opening of Standard 3 lessons

Lesson	1	2	3	4
Range	300 to 600	400 to 600	450 to 600	500 to 600
Skip value	10	10	10	10 then 5

All the examples used during the tasks that followed the skip counting involved the sum of two numbers written in place-value layout under the place-value headings H, T, and O. For

instance, $442 + 106$ was presented on the chalkboard as

$$\begin{array}{r}
 \text{H T O} \\
 442 \\
 + 106 \\
 \hline
 \hline
 \end{array}$$

The list of examples used across the four lessons has been presented in Table 4-20. It can be noted in Table 4-20 that the Standard 3 teacher was more consistent in the number of examples used, using four examples during each lesson.

Table 4-20: Examples used across the four Standard 3 lessons

Type of Task	Lesson 1	Lesson 2	Lesson 3	Lesson 4
Teacher and whole class	$442 + 106$	$541 + 27$	$346 + 138$	$327 + 118$
Group work		$412 + 167$	$263 + 129$	$519 + 6$
Individual work	$574 + 2$, $207 + 112$, $351 + 36$	$425 + 42$, $361 + 128$	$318 + 242$, $375 + 17$	$376 + 19$, $126 + 439$

As shown in Table 4-20 the first problem for each lesson was discussed with the whole class as an example and solved on the chalkboard (see Figure 4-4). Except for Lesson 1, the next problem was given as group work, and the last two problems were mostly given as individual work.

The rationale for the Standard 3 teacher's choice of tasks and examples

The teacher explained during an interview that both her choice and sequencing of tasks and examples were done on purpose.

The rationale for the teacher's selection of tasks

The reasons for using the various types of tasks observed during the four lessons were outlined by the teacher in Excerpt 4-26.

87. R: So, is there any reason for following the sequence in this way, so, that after the introduction you should have an example, followed by group work, and then individual work?
88. T: Yes. During the introduction, we remind the learners what we have done before. For example, they may have counted numbers before, so, those numbers they counted before would now be used during addition.
-
93. T: Now, regarding the example, we want to let the learners know that when carrying out addition they should be adding in proper order. For example, when they have ones, they have to add them separately, if they have tens should also be added with tens only, and hundreds should also be added with other hundreds.
94. R: Mhm.
95. T: Yes. Now, when we give them group work, the aim is that, if there is a learner who didn't understand the example that was given, other learners within the group might have understood it properly. So, the ones who understood would help that one individual to get it.
96. R: Mhm. Now I get it; assuming that the others explained to their friend, not so?
97. T: Yes.
98. R: Mhm.
99. T: Regarding individual work, our aim is to check, "has every learner understood the example that was given?"
100. R: Okay.
101. T: Sure.
102. R: Mhm.
103. T: When doing correction, our aim is that if there is a learner who did not understand, they should now get it clearly. So, if they made a mistake somewhere, they could correct it after knowing that it was supposed to be written like this.

Excerpt 4-26: Reasons for using various task types during the lessons.

From the teacher's explanation in Excerpt 4-26, each task built on the previous one. For instance, the group work was meant to let peers who understood the whole-class example to help their friends who did not understand. The teacher used individual work to get feedback on the learners' understanding. If some still faced difficulties, they were helped during the discussion of the individual work with the whole class.

The rationale for the teacher's selection of examples

The teacher explained that there was some intentional selection of examples across the four lessons. She said that the examples used during Lesson 1 were selected in such a way that they would not require regrouping. The teacher also explained that she checked examples to use based on their closeness to the teachers' guide. She indicated that this was necessary because there was more than one version of the learners' mathematics textbooks being used, hence she had to pick the examples from the version whose page numbers matched with the teachers' guide. The teacher also explained that examples were selected based on complexity, based on the number of digits for the addends. Instead of just giving the learners simple problems, she could also select some challenging problems to assess her learners' competence.

4.5.3 Mediating artefacts

This section discusses the nature of artefacts observed during the Standard 3 lessons and their major roles.

Nature of artefacts used in Standard 3

During the four Standard 3 lessons, the major artefacts used were the spike abacus and framed counters.

Spike abaci

The general functionality of the spike abacus was illustrated in the teachers' guide as shown in Figure 4-66.

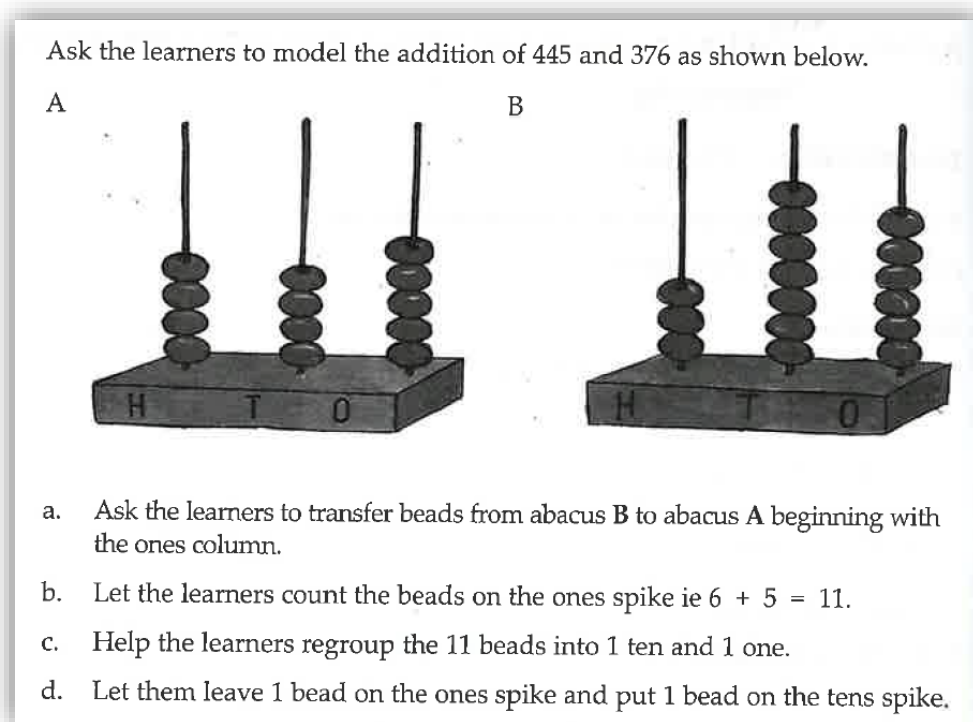


Figure 4-66: Snippet of the first four instructions in the teachers' guide for modelling the addition of two numbers using two spike abaci (Source: Malawi Institute of Education, 2013, p. 44).

During all the four lessons, the spike abaci were used for working out examples done by the teacher with the whole class as well as those done in groups. Most learners seemed to solve the problems given as individual work using other means. However, when learners were informed about individual work during Lesson 1, they were told not to move out of their groups. A possible assumption is that the teacher wanted them to use the shared abaci for the individual work, even though the learners appeared that they did not use them.

The teacher explained during an interview that she asked her children at home to look for clay that she used for making the abaci. She also asked learners who would manage to make some abaci at their homes using a sample shown by the teacher. The learners demonstrated creativity in that they managed to come with the abaci using a variety of materials shown in Figure 4-67.

The teacher supplied counters that were used on all the abaci during the lessons. The counters were pierced bottle tops that were fitted on the spikes of the abaci.

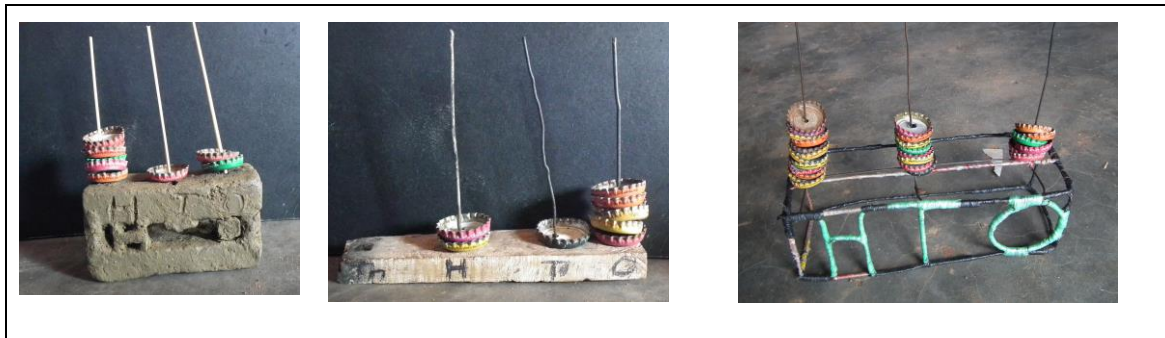


Figure 4-67: Abaci made by learners from various materials (Source: Researcher).

The abaci made from clay were frail and the teacher often reminded the learners to handle them with care. Some of the abaci were relatively small to be seen clearly from the back of the classroom (see Figure 4-68).



Figure 4-68: Relative visibility of some abaci from the back of the classroom (Source: Researcher).

Since the groups had their abaci, the teacher's instructions were easy to follow even if the teacher's abacus which was demonstrated in front of the class could not be seen clearly from the back. The teacher often went around the classroom to show the learners the resulting abacus after each major step. The teacher continued directing the learners on how to work with the abaci up to Lesson 3.

Framed counters

During the first lesson, the teacher also worked with framed counters when verifying solutions to the problems that were given during individual work (see Figure 4-69). During an interview, the teacher said that during Lesson 1, the learners had the liberty to use counters because the numbers did not require regrouping, hence counters would also do.



Figure 4-69: The Standard 3 teacher's use of framed counters during Lesson 1 (Source: Researcher).

The Standard 3 teacher's use of artefacts

The teacher used a pair of abaci for representing every stage in the process leading to finding the solution to the given problems as outlined in Figure 4-66. When introducing the use of abaci during Lesson 1, the teacher distributed specific colours of the counters to be used for a particular place-value. The teacher explained in an interview that she did this to familiarise the learners with the relative positions of the place-values. From the second lesson onwards, the colours were not followed any more. The teacher explained that she used the colours to familiarise the learners with the place values of the spikes, and felt that colour coding would no longer be necessary after the learners have mastered the positions.

During the last task of Lesson 1, verification of the solutions found during individual work was done using counters. The teacher asked the learners who were coming to solve problems in front of the class to bring their counters (see Figure 4-70).



Figure 4-70: Learners' use of framed counters for verifying solutions during Lesson 1 of Standard 3 (Source: Researcher).

From Lesson 2, the learners asked to come to the front of the classroom to solve problems were asked to bring their two abaci with them (see Figure 4-71).



Figure 4-71: Learners' using abaci for verifying solutions in front of the classroom (Source: Researcher).

During counting led by a learner in front, the class often mentioned the numbers ahead of the one who was leading. In one instance during Lesson 2, the learners counted faster up to three times in a sequence until the teacher reminded them to wait for the one handling the counters.

After a learner in front had finished composing the representation of a number using an abacus, the teacher would ask them to show the abacus to the whole class.

There was some level of abstraction that was associated with the use of the abacus. Although the teachers' guide outlined the process as shown in Figure 4-66, it does not offer any explanations. The teacher had to think of a convincing explanation for the representation of 10 counters with a single counter when the sum exceeds 10 on one spike of the abacus. In the end, the teacher devised her novel way of representing 10 counters in the abacus with a single counter that seemed to work for her class. When the sum of counters on corresponding spikes exceeded 10, the teacher asked the learners to remove 10 counters and raise one counter as a representative of the 10 (see Figure 4-72).



Figure 4-72: Raising a counter representing 10 (Source: Researcher).

During an interview, the teacher also commented that raising the counter representing a 10 also helped in attracting the attention of those who might have been distracted by something else during the process of counting.

The challenge that was observed with the use of the abacus was its orientation relative to the one reading the represented number. There was a possibility of flipping the abacus, and when the abacus was flipped the number changed. This was observed during Lesson 4 when a learner using an abacus in the front of the classroom had inadvertently flipped it. The teacher quickly noticed and helped in setting the abacus to the correct orientation. During the outset of this lesson (Lesson 4), the teacher asked the learners to check the place-value marks on the abacus. In Lesson 1, inadvertent flipping was counter-checked with colour coding of the counters belonging to a specific place value.

The rationale for the Standard 3 teacher's use of artefacts

During an interview, the teacher explained that the use of artefacts matched the complexity of the problems being worked on. She said that the use of counters would be challenging for problems involving regrouping, which were easier handled with the abacus. Besides usability, it appears that another goal of the lessons was also to teach the learners the concept of the abacus as a mathematical object in itself (see Excerpt 4-27).

200. R: So, what's the role of the abacus?

201. T: Their main goal was for the children to know: "What is an abacus, and how do we use the abacus?"

Excerpt 4-27: The goal of teaching the abacus as a mathematical object.

4.5.4 Mediating inscriptions

The teacher presented all the examples solved during the four lessons using chalkboard inscriptions.

Nature of inscriptions in Standard 3

The inscriptions used across the lessons were two-fold: The inscriptions for presenting the task to be worked out, and inscriptions for showing the method for arriving at the solution (see Figure 4-73).

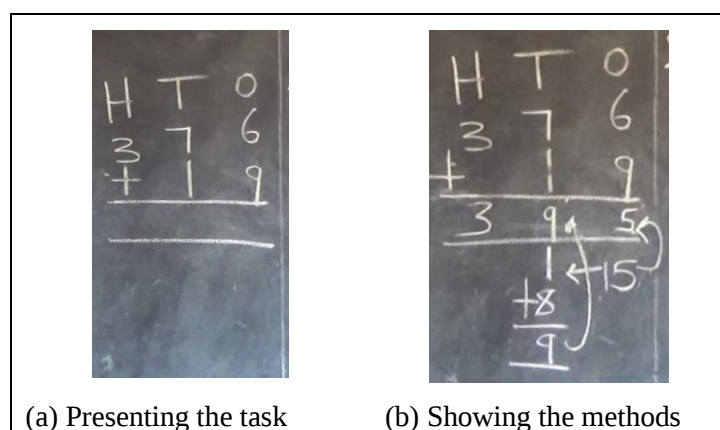


Figure 4-73: Nature of inscriptions in Standard 3 (Source: Researcher).

The inscriptions for showing the procedure for working out the problems included arrows indicating where the shown digits on the final answer originate from, as shown in Figure 4-73 (b). During Lesson 1, the teacher also used a sketch to depict how the abacus holding the answer for $442 + 106$ would appear. The sketched abacus and the represented number were placed side-by-side (see Figure 4-74).

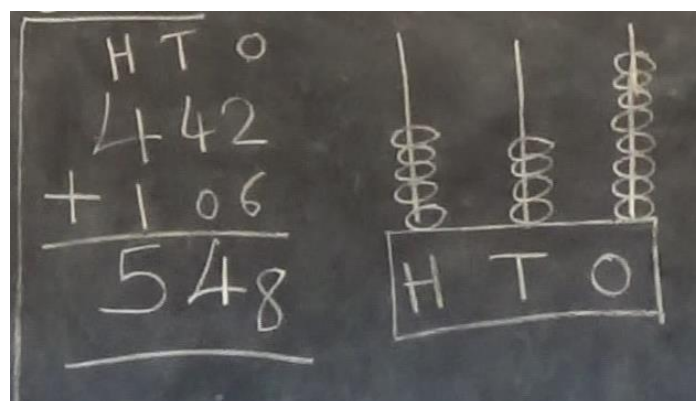


Figure 4-74: Sketched representation of 548 on an abacus (Source: Researcher).

The Standard 3 teacher's use of inscriptions

The teacher always asked the learners to read the problem first before starting to work it out with the class. Even learners who were invited to come to the chalkboard were asked to read first before working out the problem. The learners became used to the routine of starting with reading the statement so much so that during Lesson 2, when a learner started working on a problem in front before reading it, one learner in the classroom shouted to the fellow learner in front: "You did not read!" After obtaining the answer, the teacher also asked the learner to read out the resulting entire addition statement.

The rationale for the Standard 3 teacher's use of inscriptions

Unlike other teachers who erased inscriptions for a completed task, the Standard 3 teacher left all the chalkboard work for all the tasks done that day, as shown in Figure 4-4. The inscriptions provided a means for recording the method for generating solutions and showing mathematical connections, as discussed in the next section (4.5.5).

4.5.5 Mediating talk and gesture

The Standard 3 teacher's talk and gesture was centred around the use of abaci when adding the given numbers.

Mediating talk and gesture for providing methods for generating solutions

During Lesson 1, the teacher started the addition of numbers using a pair of abaci by reminding the learners the basic principles of place-value addition, such as always starting from the ones. Since the learners had done place-value addition for 2-digit numbers in Standard 2, they were sometimes quick to solve the given problems without using the abacus. In Excerpt 4-28 for instance, when working out $442 + 106$ during Lesson 1, the class quickly gave 8 as the sum of 2 and 6 on the ones' spike before they had used the abacus. The teacher had to reject the quick offer and redirected them to count on the abacus.

273. T: Fine. Now, you should count along the ones, on the abacus. You have found that two plus six, is how many? Can you count?

274. C: Eight!

275. T: No! You should count what you have in the abacus.

...

280. C: *[Count as the teacher pushes the counters along the ones, spike on her abacus]* One, two, three, four, five, six, seven, eight!

281. T: So, what number should we put on the first spike?

282. C: Eight!

Excerpt 4-28: Redirecting learners to use the abacus.

The teacher explained in Excerpt 4-29 that the major reason for redirecting the learners to the abacus was for them to follow the method she used in the first example and discussed with the class.

193. T: I wanted the children to use the method that I used in the example since we were using abaci. So, when children were working out the problem, they were also supposed to use an abacus first, not just rushing to write.

Excerpt 4-29: The teacher's reason for redirecting the learners to the abacus.

As shown in Figure 4-73, the method was recorded on the chalkboard using inscriptions containing arrows showing how the values were obtained with the regrouping algorithm.

However, during individual work the majority of learners seemed to work out the solutions using other methods and not using the abaci. This might be because the abaci were shared or because the learners knew more efficient ways of solving the problems.

Mediating talk and gesture for building mathematical connections

When working out the solutions, the teacher always asked why something had to be done as it was done. For instance, during Lesson 3 the teacher asked: "Why is it that along the ones, we did not write 12?". She repeated this question until she obtained a plausible answer from one of the learners. By asking "why", the teacher wanted the learners to make connections between

what they were doing and the properties of numbers. For instance, the teacher defined the addition with zero on as being equivalent to “not adding anything on the abacus”.

Use of language

Oftentimes, the teacher had to deal with vocabulary limitations of the language of instruction, Chichewa. For instance, if she was using English, there were instances where the teacher intended to refer to the digits in a given number. She used the same term “nambala” that is used for number to refer to individual digits along their respective place-values. Excerpt 4-30 shows a scenario when the teacher had just started to work out $541 + 27$ with the class during Lesson 2. In the episode, the teacher was starting to build the first abacus for 541 and wanted to ask the class to mention the digit along the ones:

T: Fine. <i>[Gives an abacus to a group] [Talks to the class]</i> So, let us start with the first abacus, alright? 42. T: Yes! Along the ones, how many numbers are there? 43. C: Two! 44. T: No! The first number is this one, alright? <i>[Points at 541]</i> 45. C: Yes! 46. T: So, along the ones, how many things are there? <i>[Points at 1 in 541]</i> 47. C: One! 48. T: There is one, alright? 49. C: Yes!
--

Excerpt 4-30: Use of “number” to refer to “digit”.

The teacher basically wanted to say: “What is the value of the digit along the ones for the first abacus?” After noting the vocabulary limitation, the teacher changed the question from “how many numbers” to “how many things”.

Mediating talk and gesture for advancing learning connections

During the introductory skip counting episodes, the class was asked to count in intervals of 10 up to 600 with different starting values between 300 and 500. The class counted properly in

10s but errors started being noticed when individual learners were asked to count with the skip value changed to 5 during Lesson 4. In Excerpt 4-31, the learners had been asked to count from 500 to 600. One learner (L2) in Utterance 12 did not even attempt to read. Another learner (L3) got lost after counting three steps from 500.

- | |
|--|
| 11. T: You will be adding five. |
| 12. L2: <i>[Fails to proceed]</i> |
| 13. C: <i>[Raise hands]</i> Madam! Madam!... |
| 14. T: Yeah! That boy! |
| 15. L3: Five hundred five, five hundred ten, five hundred fifteen, five hundred sixteen, five hundred seventeen... |
| 16. T: No! You will be adding fives. |

Excerpt 4-31: One learner's skip counting in 5s.

Oftentimes, when an incorrect offer was given, the teacher outrightly said, “no!” (see Utterance in Excerpt 4-31 and Utterance 130 in Excerpt 4-32).

- | |
|---|
| 129. T: <i>[Talks to L11]</i> ... In fifteen, how many fives are there? ...ones... in fifteen, how many are there? She says that it contains one ten and how many ones? |
| 130. L11:Two. |
| 131. T: No! |
| 132. C: <i>[Raise hands]</i> Madam! Madam!... |

Excerpt 4-32: Teacher saying “no” to a learner's offer.

The teacher used repetition when communicating fundamental steps for working out a problem. During Lesson 2, the teacher repeated up to four times when communicating instructions that required some precision for the whole class to arrive at the same answer.

4.5.6 Summary of the usage of mediational means in Standard 3

The Standard 3 teacher's use of tasks and examples was consistent across all the lessons. The lessons started with skip counting, followed by one example done with the whole class. Thereafter, the teacher gave one example that was worked out in groups and two examples done individually by learners in their notebooks and marked by the teacher. The use of

examples was structured to start with numbers that did not require regrouping during Lesson 1, to numbers that required regrouping during the subsequent lessons.

Regarding the use of artefacts, the teacher worked with pairs of spike abaci representing the two addends being worked on. Each digit corresponded to a spike in the two abaci being used to perform the calculation. The teacher initially used colours to indicate counters belonging to a particular place-value during Lesson 1 and shifted to no use of colour during the subsequent lessons after the learners had been familiarised with the abacus. The teacher devised a method for representing a group of 10 counters with a raised counter when moving it to the next place value. The abaci were only used during tasks done by the whole class and in groups because, during individual work, learners solved the problems silently using other strategies. The use of framed counters was only noticed during the last episode of Lesson 1.

The teacher worked with chalkboard inscriptions for presenting examples to be worked out and for showing the method followed to arrive at an answer. The method was shown using arrows as well as numbers written below the given problem.

The Standard 3 teacher's talk for generating solutions focused on reminding learners the order followed when adding numbers aligned according to the place-values of their digits. The teacher's mediating talk for building mathematical connections centred on showing the connections within an example, visually displaying them with arrows. There were some opportunities for advancing learning connections from learner's wrong offers.

4.6 Use of mediational means in Standard 4

Four lessons were observed in Standard 4. Out of the four lessons, three were taught consecutively during the eighth week, while the fourth lesson was taught during the 11th week of the first term.

4.6.1 An overview of Standard 4 lessons

Lesson 1

In Episode 1, the teacher asked learners to workout $60 + 11$, $70 + 30$, $82 + 12$ and $43 + 7$ mentally. In Episode 2, the teacher put a chart-paper on the chalkboard with $3353 + 2122 + 2113 + 1211$ for learners to solve individually and marked them. The teacher verified the solution with the whole class using counters. In Episode 3, the teacher asked the class to locate two problems in their textbooks ($1432 + 4223$ and $4103 + 3242$) and work them out in pairs. This was followed by verification of the solutions on the chalkboard by two learners. During the fourth episode learners were given reference to four problems in their textbooks to work them out individually and thereafter marked by the teacher. During the last episode, the teacher gave two problems ($56 + 20$ and $14 + 4$) to be solved mentally. The first lesson has been presented in a lesson graph in Appendix 17.

Lesson 2

The second lesson started with a review of writing place-value headings and reading some numbers (5330, 8679, 7843, 6256) in Episode 1. This was followed by presenting $3165 + 2314 + 3723$ on the chalkboard and worked out by the teacher and the whole class during Episode 2. During Episode 3, the teacher gave problems to groups and verified the solutions by swapping the group solutions so that the groups could check each other's work. In Episode 4, the teacher gave two problems ($4102 + 1893 + 2016$ and $2431 + 1007 + 3445$) to be solved individually and marked them. The solutions were verified by two learners on the chalkboard. The teacher gave one more problem ($1345 + 3316 + 1452 + 1232$) as homework. The progression of Lesson 2 has been presented in the lesson graph shown in Appendix 18.

Lesson 3

Lesson 3 started with the teacher asking learners to work out $1685 + 1298$ and $6234 + 1398$ mentally during the first episode. The teacher then worked out $1450 + 4128 + 2323 + 1979$ with the whole class during Episode 2. In Episode 3, the teacher distributed chart papers with $5898 + 524$, $5865 + 4075$, $8256 + 1485$, $1549 + 1286$, $1272 + 1764 + 4528$, $3658 + 1278$, $6037 + 1683$, $7289 + 0394$, $3644 + 1789$ to groups. Each group worked out their problem and pasted their chart paper on the wall and presented their solution to the whole class. In Episode 4, the teacher gave $2423 + 3434 + 2708 + 1195$ as individual work and marked the notebooks. The teacher then asked a learner to verify the solution on the chalkboard. The lesson has been presented in a lesson graph in Appendix 19.

Lesson 4

The teacher started the first episode by writing a word problem on the chalkboard that was reduced to $2375 + 2240 + 1850$ and worked out the solution with the class. In Episode 2, the teacher distributed word problems written on chart papers for learners to work them out in their groups. The groups posted their solutions on the wall and presented them to the whole class. In the third episode, the teacher wrote a word problem on the chalkboard and asked learners to work it out in their notebooks. The teacher verified the solution with the whole class and asked those who made mistakes to do corrections. The lesson graph in Appendix 20 shows how Lesson 4 progressed.

4.6.2 Mediating tasks and examples

Nature of tasks and examples observed in Standard 4

In Standard 4, the tasks involved adding numbers with a sum not exceeding 9,999. Whereas the numbers being added from Standard 1 to Standard 3 had two addends, examples used in Standard 4 had up to 4 addends.

Types of tasks

Across the four lessons, the teacher presented the introductory tasks in a variety of ways, such as solving mental addition problems, writing place value headings, and reading numbers on pieces of paper. The teacher introduced Lessons 1 and 3 by asking learners to add some pairs of numbers mentally. The mental problems that the teacher gave during the introduction of Lesson 1 had smaller addends than those given during Lesson 3. Lesson 2 was introduced using papers that had various numbers to be read by learners. The last lesson started with a word problem solved by the teacher and the whole class.

In addition to introducing lessons in a variety of ways, the Standard 4 teacher also presented tasks differently. Shortly after the introductory mental addition activity during Lesson 1, the learners were also given another problem to solve individually in their notebooks and was marked by the teacher (see Excerpt 4-33). After the learners had completed the task individually, the teacher asked the learners to solve two problems in pairs followed by some more individual work. During the closing session of Lesson 1, the teacher asked the class to find the sum of small addends mentally. During Lesson 2, the teacher used some of the principles of the jig-saw teaching method (Aronson & Bridgeman, 1979) during group work (see Figure 4-75). With the jig-saw method, the teacher gave different problems written on chart papers to different groups. After the groups had finished working out the solutions, she swapped groups to study the work done by the other groups and verify if the solutions were correct.



Figure 4-75: Groups studying the work done by the other groups during Lesson 2 (Source: Researcher).

Lessons 3 and 4 were dominated by tasks that were done in groups, then presented to the whole class by a learner in the front of the classroom representing the group. There was a slight variation between the group work given during Lesson 3 and Lesson 4. During Lesson 3, each group was given a unique problem while during Lesson 4, two groups were given the same problem. Because of this, when two groups found the same answer during Lesson 4, the class considered that as verification and therefore did not spend time working the problem as whole class to verify. As such, less time was spent by the group representatives during Lesson 4 compared to Lesson 3 (see Figure 4-76).

Just like the Standard 2 and Standard 3 teachers, the Standard 4 teacher also collected unmarked notebooks when she was about to revise the individual work with the whole class. Learners that had incorrect solutions were asked to rewrite the tasks to be marked the next day.

Duration of tasks

Except for the first lesson, the Standard 4 lessons were completed in over an hour (see Figure 4-76). The longest lesson, that focused on solving word problems, lasted 1 hour 25 minutes.

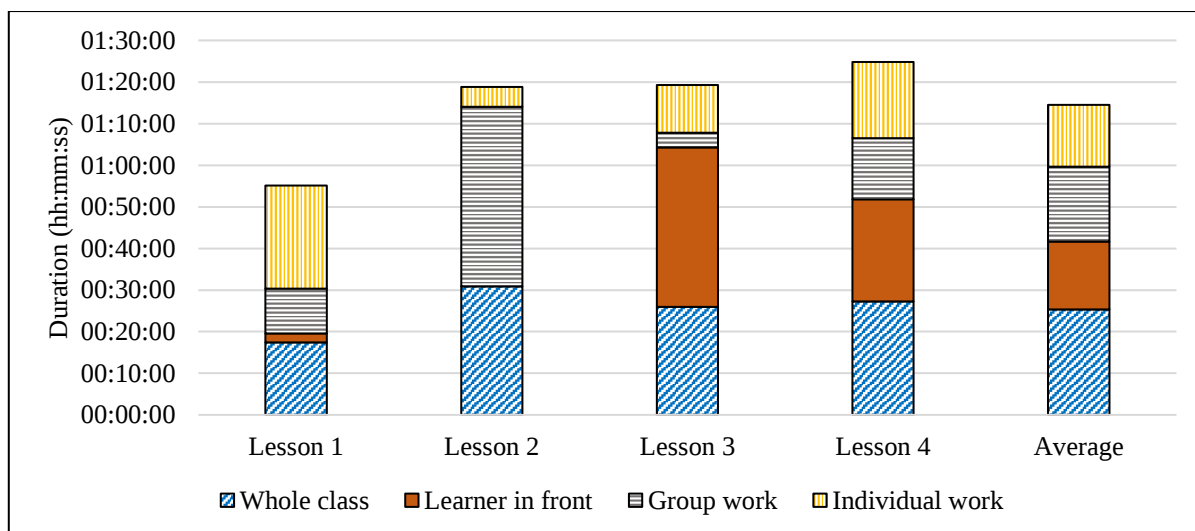


Figure 4-76: The time spent on various types of tasks across the four Standard 4 lessons (Source: Researcher).

It can be seen in Figure 4-76 that the time spent on various types of tasks was not much consistent for the Standard 4 teacher across the four lessons. However, on average, the teacher's relative use of each task type appeared to be balanced to some extent (see Figure 4-77).

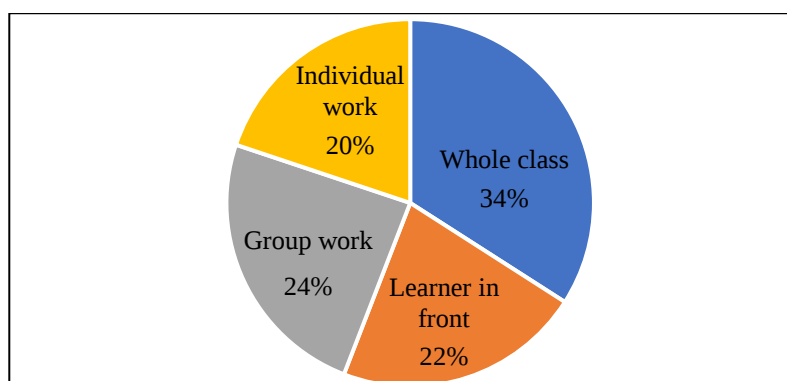


Figure 4-77: Average duration of task types in Standard 4 (Source: Researcher).

Lesson sequence

As highlighted previously, the Standard 4 lessons did not follow a definite pattern as was the case with Standard 3. Despite this variation, the teacher stated during an interview that her lessons were generally structured to start with an example, followed by group work, then

individual work, which was revised with the class and sometimes followed by homework. This structure was not different from the one mentioned by the other teachers.

Examples used in Standard 4

The examples used during the Standard 4 lessons have been presented in Table 4-21.

Table 4-21: List of examples used in Standard 4

	Lesson 1	Lesson 2	Lesson 3	Lesson 4
Episode 1	$60 + 11, 70 + 30, 82 + 12, 43 + 7$	5330, 8679, 7843, 6256	$1685 + 1298, 6234 + 1398.$	$2375 + 2240 + 1850$
Episode 2	$3353 + 2122 + 2113 + 1211$	$3165 + 2314 + 3723$	$1450 + 4128 + 2323 + 1979$	$3855 + 1976, 4928 + 4072, 1550 + 1350 + 1050 + 1400, 3442 + 2307 + 2850 + 1328, 1460 + 2955 + 1178 + 3720.$
Episode 3	$1432 + 4223, 4103 + 3242$	$3383 + 1226, 1905 + 2589 + 1357, 1986 + 1748 + 1637, 2083 + 3914 + 1001, 2578 + 2057 + 1193.$	$5898 + 524, 5865 + 4075, 8256 + 1485, 1549 + 1286, 1272 + 1764 + 4528, 3658 + 1278, 6037 + 1683, 7289 + 0394, 3644 + 1789$	$1255 + 1785 + 2370.$
Episode 4	$2100 + 2232 + 3135, 1031 + 4315 + 1222, 2132 + 1326 + 4301 + 1210, 2414 + 2220 + 3112 + 1133$	$4102 + 1893 + 2016, 2431 + 1007 + 3445, 1345 + 3316 + 1452 + 1232$	$2423 + 3434 + 2708 + 1195$	
Episode 5	$56 + 20, 14 + 4$			

As mentioned at the beginning of this section (4.6.2), among the examples shown in Table 4-21, the examples in Episodes 1 and 5 of Lesson 1 were verbally presented by the teacher and learners solved them mentally, while the numbers in Episode 1 of Lesson 2 had to be read out by the learners. The problems were mostly presented in place-value layout under the place-

value headings Th, H, T, and O (see Figure 4-84). All the examples done during Lesson 4 were presented as word problems selected from the learners' textbook (see Table 4-22).

Table 4-22: Word problems used in Standard 4 Lesson 4

Episode	Example
Episode 1 (Whole class example)	At Dambo School there are 2,375 learners, at Thengo School there are 2,240 learners, while at Macheke School there are 1,850 learners. How many learners are there altogether?
Episode 2 (Group work)	<u>Groups 1 and 2:</u> Chafika Village planted 3,855 trees, while Chatha village planted 1,976 trees. How many trees were planted altogether? <u>Groups 3 and 4:</u> On the first day, people moulded 4,928 bricks. On the second day, they moulded 4,072 bricks. How many bricks were there altogether? <u>Groups 5 and 6:</u> Football supporters came to a sports ground as follows: 1,550 boys, 1,350 girls, 1,050 women, and 1,400 men. How many supporters came altogether? <u>Groups 7 and 8:</u> During elections, Mrs Dziko got votes from four locations as follows: 3, 442; 2,307; 2,850 and 1,328. How many votes did she get altogether? <u>Groups 9 and 10:</u> Takula Village has 1,460 women, 2,955 boys, 1,178 men and 3,720 girls. How many people are there in the village altogether?
Episode 3 (Individual work)	Ngozo has 1,255 chickens, Chesupuni has 1,785 chickens, while Najere has 2,370 chickens. How many chickens are there altogether?

The contexts highlighted in the word problems included activities that the learners were familiar with, such as moulding bricks, planting trees, and raising chickens. The problems are all framed in the same manner and are structured to be worked out using the 'combine and count all' strategy of addition.

The rationale for the Standard 4 teacher's choice of tasks and examples

The rationale for the teacher's selection of tasks

Regarding the use of mental addition activities observed during Lesson 1 and Lesson 3, the teacher indicated in Utterance 55 of Excerpt 4-33, that these sessions acted as a warm-up

activity. The teachers' guide also suggested giving the learners some addition problems to be solved mentally by the learners.

1. T: I want you to find for me answers to these problems: 60 plus 11. 60 plus 11.
...
6. L1: 71.
...
9. T: 70 plus 30. 70 plus 30.
...
12. L2: 100.
...
27. T: Aha, fine. What about 82 plus 12? 82 plus 12?
....
38. L7: 94.
41. ... Another problem: 15 plus 15, what is it altogether?
....
44. L8: 30.
...
47. T aha. Another problem: 43 plus 7, what is it altogether?
...
50. L9: 50.

51. T: [Points to L10] Eh?
52. L10: 50.
....
55. T: Aha! That was a brain awakening session to activate your heads.

Excerpt 4-33: Mental activity during Lesson 1.

During Lesson 3, the teacher commented in Excerpt 4-34 that familiarity with mental problems would help the learners to not only be able to solve problems when they have somewhere to write.

40. T: Yes! We should not just be used to writing in the notebook or on the chalkboard. No. But we should also be using the head very much. We need to be very attentive, alright?

Excerpt 4-34: Rationale for mental problems during Lesson 3.

During an interview, the teacher explained that group work helped her in content coverage. She indicated that her large number of learners implies that many groups were also formed. Having more groups meant that many problems could also be assigned to the groups in a single lesson. This was implemented by asking the group representatives to explain in the same manner as if they were teaching the class. As such, a group representative was addressed as “our teacher” by the Standard 4 teacher.

As regards individual work, the teacher explained that it helped her determine the learning pace of the class. In some cases, the insights from individual work would necessitate repeating a lesson. As for homework, the teacher explained that it helped the learners not to forget the concepts when they wrote again at home, where they could also be assisted by a capable relative.

The rationale for the teacher’s selection of examples

Regarding the selection of examples, the Standard 4 teacher worked closely with the learners' textbook, ensuring that the learners complete all the presented problems. From Lesson 1, the teacher asked the learners to follow her from their textbooks. The teacher had to first establish the page numbers with the class because the learner’s textbooks were in two editions. For instance, the work that was done during Lesson 1 was on page 35 in one edition of the learners’ textbooks while in the other edition the same work was on page 37.

4.6.3 Mediating artefacts

Nature of artefacts

The teacher used loose counters in a plate when working out tasks with the class during the first two lessons (see *Figure 4-78*).



Figure 4-78: Use of loose counters during Lesson 1 (Source: Researcher).

During the last two lessons, the teacher used framed counters when working out problems with the whole class (see Figure 4-79).



Figure 4-79: Use of framed counters during Lesson 4 (Source: Researcher).

The teacher also used pieces of paper with prewritten numbers for learners to read out during the introduction of Lesson 2 (see Figure 4-80).

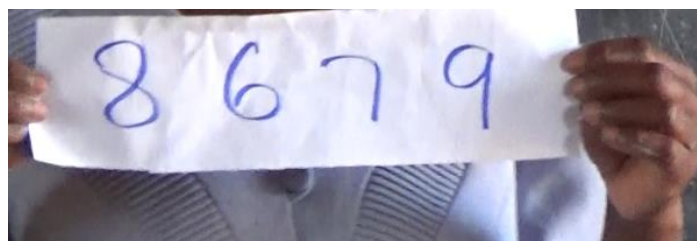


Figure 4-80: Prewritten papers with numbers during Lesson 2 (Source: Researcher).

During Lessons 3 and 4, the teacher also used prewritten problems on chart-papers for learners to solve in groups (see Figure 4-81).

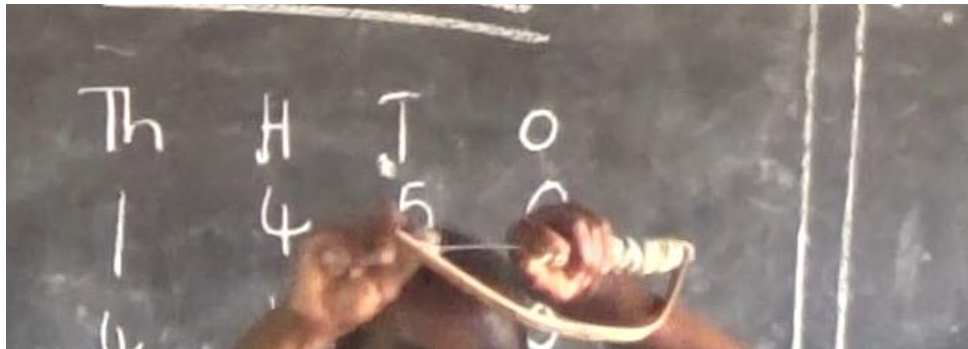


Figure 4-83: Adding a zero using framed counters during Standard 4 Lesson 3 (Source: Researcher).

During Lesson 4, adding a zero with counters was represented by not doing any action, as shown in Excerpt 4-35 during Lesson 4.

57. T: So, we are being asked to add zero. How should we do this?
 58. C: We will not put anything!
 59. T: Eh?
 60. C: We will not put anything!
 61. T: We will not put anything?
 62. C: Yes!
 63. T: Meaning that it will just be the same 5?
 64. C: Yes!

Excerpt 4-35: Adding zero during Standard 4 Lesson 4.

The rationale for the Standard 4 teacher's use of artefacts

The teacher said that she used artefacts to enhance her learners understanding. Regarding the use of counters, the Standard 4 teacher said that: “If we just explain to a child without using any object, it is sometimes difficult for them to understand clearly. But when they use a real object, they can even practise using it when alone.” However, when learners were invited to the front to work out the solution to a problem, they were not seen using counters as the teacher did.

During an interview, the teacher explained that the use of charts posted on the walls of the classroom extended the lesson time indirectly. The charts allowed learners to see how their fellow learners worked out a particular problem during their own time, such as break time. The Figure 4-84 below shows a learner presenting work by Group 7 during Lesson 3. The group's chart is placed on the wall next to the group's previous work from Lesson 2, which was still available during Lesson 3 for all to see.

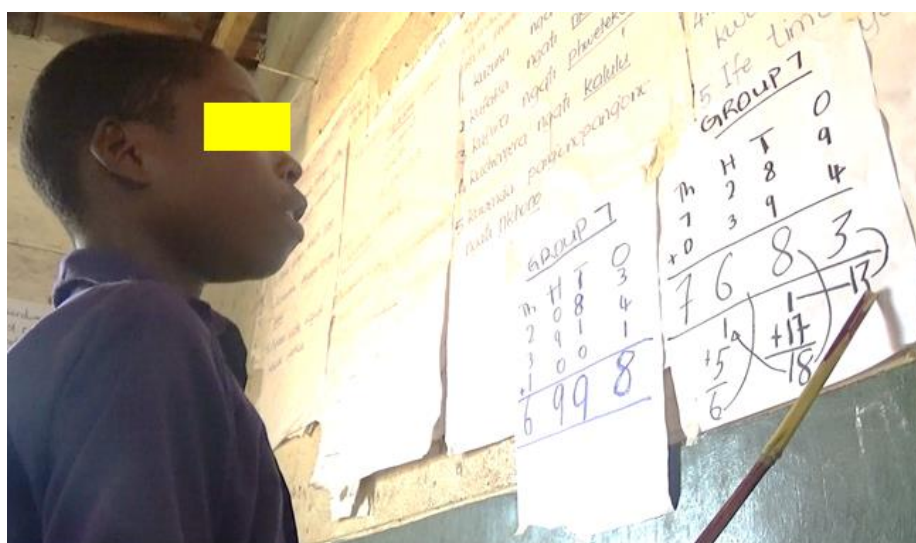


Figure 4-84: A learner presenting group's work written on a chart paper pasted on the wall next to group's work from previous lesson (Source: Researcher).

Regarding the use of an abacus in Standard 4, the teacher said that it is only used during lessons directly related to its use, otherwise, only counters are used.

4.6.4 Mediating inscriptions

Nature of inscriptions in Standard 4

The inscriptions observed in Standard 4 were the same as those used in Standard 3. Some inscriptions were for presenting tasks and examples while some were for showing the method for generating solutions (see Figure 4-85).

The Standard 4 teacher's use of inscriptions

The teacher used chalkboard inscriptions for presenting tasks and examples. After finding the solution to the first problem with the class on the chalkboard, the teacher asked learners to come forward and work out the solutions of the rest of the problems. Figure 4-85 shows a learner working out the solution to one of the two problems that were given as individual work during Lesson 2.



Figure 4-85: Nature and use of chalkboard inscriptions in Standard 4 (Source: Researcher).

The rationale for the Standard 4 teacher's use of inscriptions

The teacher explained in Excerpt 4-36 that her inscriptions acted as a model for the learners to follow, hence she had to write more legibly.

142. T: When writing an example, I ask everyone to look at the chalkboard and see what I'm doing and say: "so you should also write exactly in the same way as I have written." so, when you are writing on the chalkboard or the chart, you also try to write very clearly so that the learners can also follow....

Excerpt 4-36: The role of teacher's inscriptions.

The teacher's explanation in Excerpt 4-36 explains why she used half of the chalkboard space when working out $3165 + 2314 + 3723$ shown in Figure 4-84.

4.6.5 Mediating talk and gesture

The Standard 4 teacher had some aspects of her mediating talk and gesture that were unique.

The Standard 4 teacher's mediating talk and gesture for providing methods for generating solutions

Just like the Standard 3 teacher, the Standard 4 teacher reminded the learners during Lesson 1 where to start from when performing place-value addition. After discussing that it should be along the ones, the teacher went-on to emphasize on the side from the perspective of the teacher as well as the perspective of the learners (see Excerpt 4-37).

- | |
|---|
| 108. T: The ones, alright? From the position that you are seated, you are supposed to start adding from which hand? |
| 109. C: The right-hand side. |
| 110. T: The right-hand side, alright? |
| 111. C: Yes! |

Excerpt 4-37: Emphasizing the relative positions of where to start adding.

To lighten the mood when working with the regrouping algorithm, the teacher sometimes personified the numbers. For instance, in Utterance 197 of [Excerpt 4-38](#) from Lesson 2, the teacher was talking to the digit 1 to move to the next place value.

- | |
|---|
| 195. T: <i>[Points to L17]</i> What should we do with one? |
| 196. L17: That one should also go to the hundreds! |
| 197. T: She is saying that one should go to the hundreds. Fine. We should take one. We are taking this one: “You one, let’s go there!” We should take one and move it to the hundreds <i>[Draws an arrow joining the digit 1 from 10 below the bottom bar to a new position under hundreds below the bars where she writes 1]</i> . Mhm. Let us find out the numbers that we have been given along the hundreds. |

Excerpt 4-38: Adding humour to the regrouping algorithm.

During group work, the teacher emphasized that the learners should also write down the method on their chart-papers.

When working out the first addition word-problem during Lesson 4, the teacher discussed with the class how to approach such problems. The first thing discussed with the class were the clues in the word problem that would make them conclude that it was an addition problem, such as the phrase, “how many are there altogether?” After identifying that it was an addition problem, the numbers were isolated from the problem and arranged in place-value layout. After finding the sum, the teacher went back to the original word problem to express its answer.

Counting all

As stated in section 4.6.3, the teacher used the count-all method when generating solutions using artefacts. In Excerpt 4-39 from Lesson 2, a learner quickly gave 12 as the answer for $11 + 1$, but the class went on with unit counting up to 12.

231. T:	So, we are saying that what should be added to eleven?
232. C:	One!
233. T:	We should add one, alright?
234. C:	Yes!
235. L21:	Twelve!
236. T:	Fine. So, let us add all of them together to find out how many they are altogether. Let us count!
237. C+T:	<i>[Count as the teacher puts back the counters into the plate]</i> One, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve!
238. T:	We’ve got twelve now, alright?

Excerpt 4-39: Unit counting in the count-all strategy for $11+1$.

Standard 4 teacher’s Mediating talk and gesture for building mathematical connections

The teacher made connections within examples using unique approaches. As stated in the previous subsection, the connections within examples were sometimes achieved by personifying the inscriptions shown in Figure 4-85.

Just like the Standard 3 teacher, the Standard 4 teacher also enhanced connection within examples by asking “why” something had to be done the way it was done (see Excerpt 4-40 from Lesson 3).

- | |
|---|
| 118. T: Is it not possible just to take the whole twenty and write it here? |
| 119. C: No. |
| 120. T: What is the reason? |
| 121. C: <i>[Raise hands]</i> Madam! Madam!... |
| 122. T: It will not be possible for what reason? |

Excerpt 4-40: Teacher seeking justification for processes.

Due to being used to justifications, learners developed their plausible explanations that they gave when working out a problem in front of others in the classroom. In one instance, a learner used a popular slang word “mahanzi” for “hundreds” when presenting the method used by his group. Although the class seemed to enjoy the slang, the teacher came in to correct it with the formal word “mahandiredi”.

The Standard 4 teacher’s mediating talk and gesture for advancing learning connections

Rather than working with errors in a corrective manner, the teacher worked using a preventive approach to learners’ common mistakes.

The teacher enhanced the learners’ troubleshooting skills when doing corrections. If one learner solving a problem in front had made an error, the teacher asked those offering to correct to identify the error first, and just correct the error rather than starting all over again.

The Standard 4 teacher’s teaching style seemed to empower the learners more. The learners had to follow her carefully because at any point she would make a deliberate mistake that required the learners to quickly notice and tell the teacher what to do. Frequently, she achieved this by saying the opposite, as shown in Excerpt 4-41 from Lesson 1.

98. T: ... So, we are saying that when starting our addition, I know that we start with the thousands.
99. C: No!
100. T: No?
101. C: Yes!
102. T: We are supposed to start from where?
103. C: Along the ones.
104. T: Along the ones, alright?

Excerpt 4-41: Stating the opposite.

In Figure 4-8, the Standard 4 teacher deliberately wrote the whole 16 along the tens instead of writing 6 and moving 1 to the hundreds.

$$\begin{array}{r} 2,375 \\ + 1,850 \\ \hline 165 \end{array}$$

Figure 4-86: Teacher's deliberate mistake to capture a common error (Source: Researcher).

Soon after writing the 16 in Figure 4-86, the class objected as shown in Utterance 88 of Excerpt 4-42. Having objected, the teacher pretended to be helpless, requiring the learners to tell her what to do (see Utterance 91).

85. T: How many are there altogether?
86. C: Sixteen!
87. T: So, sixteen. You say sixteen [*Writes 16 along the tens*]
88. C: Aah! No!
89. T: No?
90. C: Yes! [*Raise hands*] Madam! Madam!...
91. T: So, what should we do there?

Excerpt 4-42: Seeking support from the class.

When finalising the addition procedure for the problem shown in Figure 4-86, the teacher captured another possible error that could come from the learners at this stage. Some learners would possibly think that the inscriptions showing the method below the problem should only be consulted when the sum of the digits along a column gave a double-digit. At this point, the class and the teacher had found 5 as the sum of the thousands (see Excerpt 4-43). In this case, even though the sum had a single digit, there was a need to go back to the algorithm to check if there was no carryover digit from the preceding steps. However, the teacher stated that there was no need to bother oneself going back to the algorithm after finding a sum that was less than 10, hence there is nothing to keep.

194. T: We have found that it is five. Here, do we have to bother with keeping anything? [*As she writes 5 under thousands between the bars on the chalkboard*] It is five, alright?
195. C: [*Crosstalk*] No! There below! Below the one! Below “equals”!
196. T: [*Erases the just written 5*] What is the reason? What is the reason?
197. C: [*Crosstalk*] We had one!

Excerpt 4-43: Capturing possible errors.

To explore learners’ thinking, the teacher still gave a chance to those who were still raising hands after a correct answer had already been given when doing mental arithmetic during the introductory episode of Lesson 1. After one learner had given 100 as the answer for $70 + 30$, the teacher still gave a chance to one more learner who gave 90 as the answer and was loudly rejected by the whole class.

Reinforcement

The teacher also used humorous ways of making positive reinforcements to the learners, in addition to the usual hand clapping styles observed in the rest of the classes. For instance, during Lesson 3, she asked the learners to make the sound of a maize-mill instead of merely clapping hands, as shown in Excerpt 4-44 that follows:

394. T: What is it altogether?
395. L38: Seven thousand seven hundred twenty.
396. T: Aha! Seven thousand seven hundred twenty. Six thousand and thirty-seven plus one thousand six hundred and eighty three, all of them together, equals seven thousand seven hundred twenty. Let us run a maize mill.
397. T: Uu-u-u-uuu! Poooh! Poo-o-oo!
398. T: I have said a maize mill! Maize mill! How does a maize mill go like?
399. C: *[As they spin their hands]* U-u-u-u-u-uuu!
400. T: Yeaah! We should now go to group number seven. The one that you did at first is a train. But I said a maize mill, alright?

Excerpt 4-44: Adding humour to positive learner reinforcement.

4.6.6 Summary of the usage of mediational means in Standard 4

The Standard 4 teacher had many aspects of her teaching that seemed to be unique compared to the other three teachers. Concerning her selection of tasks and examples, this teacher taught her learners to follow the textbook. This sometimes made it easier for her to give them work to do, as she just referred to the appropriate pages. She made use of group work to cover as much content as possible. As regards the use of artefacts, the teacher mainly worked with counters. The use of counters seemed to be more of formality because the learners were able to give the answers before the counting. The teacher used chalkboard inscriptions for presenting the method for generating solutions. She used arrows to show she worked with the place-value addition algorithm to arrive at the required answer. The teacher's talk and gesture were mostly dominated by deliberate mistakes to capture common errors and humour that made the learners follow the teacher critically while enjoying the learning. This made the learners be portrayed

as doers of mathematics, often guiding the teacher on what to do next, or telling her what ought to have been done to correct a posed mistake.

4.6.7 Chapter summary

This chapter has presented the way the four teachers worked with the four mediational means identified by Venkat and Askew (2018), that is, tasks and examples, artefacts, inscriptions, as well as talk and gesture. There were some aspects of mediation that were similar among the teachers as well as some individual differences among them. It was found that the teachers mostly selected the mediating tasks and examples from the teachers' guide, but each teacher had their own considerations for determining the number of examples to work within one lesson. While some teachers gave similar tasks to groups, others gave different tasks to different groups and thus took advantage of group work to do as many examples as possible. Since the solutions from group work were always presented and verified by the whole class, the use of groupwork enhanced content coverage. It was also found that the official time for a single lesson seemed not inadequate for the teachers to offer a whole class example, give group work, and give some individual work to mark during the lesson. The teachers resolved this challenge by combining the two periods allocated for mathematics during each school day and have a double period instead of two single periods.

The teachers worked with various types of mediating artefacts, but framed counters were the most popular. Even when other types of artefacts such as the place-value box or spike abaci were required, the teachers were also observed using these artefacts in parallel with the framed counters. Learners were often reminded to follow along the counting with their personal counters that they carried with them to school every day. The teachers also used prewritten papers for presenting tasks mostly during group work.

The teachers used the chalkboard inscriptions for presenting tasks to learners. Some inscriptions were also used to show the method that was used for generating solutions. After the teachers presented the tasks and examples on the chalkboard, much of the work was mostly completed by learners taking turns to either read the written inscriptions or write the required inscriptions.

Regarding mediating talk and gesture, the teachers used the count-all strategy for addition. It was only in two out of the 17 lessons where learners were given an opportunity to use mental addition strategies and give the answer. The teachers made connections within examples mostly by using multiple means of mediation for the same task. There were only two lessons where connections across examples were emphasized through the teacher's talk and gesture. The teachers also took advantage of learners' errors to enhance learning. Offers were always verified by the whole class and appropriate justification was given to show the correctness of the offered solution.

The assumptions and possible implications of some of the notable observations made in this chapter have been discussed in the next chapter.

CHAPTER 5

DISCUSSION OF FINDINGS

5.1 Introduction

As stated in Chapter 1, this exploratory study was aimed at investigating how teachers mediate mathematics during the early years of primary school in Malawi. In Chapter 2, it was mentioned that the study adopted the sociocultural perspective of the teacher as a mediator of learning through the use of cultural tools (Kozulin, 2003; Wertsch, 2017). Venkat and Askew (2018) identified the cultural tools for mediating learning of mathematics to children in their early years of primary school as tasks and examples, artefacts, inscriptions, as well as talk and gesture. This chapter, therefore, discusses the findings regarding the nature and use of the mediating cultural tools presented in the preceding chapter.

Section 5.2 discusses the first research question on teachers' selection of mediating tasks and examples. The section points to some of the possible reasons influencing the teachers' selection, sequencing as well as the duration of the tasks and examples. Sections 5.3, 5.4, and 5.5 discuss the second research question on teachers' use of mediating artefacts, inscriptions, talk and gesture. These sections discuss the observed usage patterns with respect to literature. The discussion of the usage of each mediational means strengthens the rationales presented in Chapter 4, thereby answering the third research question. The chapter ends with a reflection on the usage experiences of the MPM framework.

5.2 Teachers' selection of mediating tasks and examples

This section discusses the findings on the first research question.

5.2.1 Curriculum expectations

As stated in Chapter 4, the examples and tasks observed in this study mostly involved finding the sum of some given numbers. The scope of the tasks and examples that the teachers worked with was mostly guided by the curriculum expectations at the level, as shown in Table 4-2, p. 76.

The nature of the mathematics curriculum for the early years of primary school in Malawi

The curriculum presented the teaching of addition using a spiral approach. For instance, in Standard 1, the learners were supposed to work with numbers from 0 to 9 the whole academic year (Malawi Institute of Education, 2012a). The curriculum was designed in such a way that the learners had to be introduced to the numbers from 1 to 5 followed by the operations on these numbers, including ordering, addition and subtraction. Later, they would be introduced to the numbers 6 to 9 and their operations. This iterative process had to be repeated when the learners reached Standard 2, where they had to work with the numbers in the ranges 10 to 20, 21 to 50, ending with 51 to 99 (Malawi Institute of Education, 2012c). They would also be expected to do the same with numbers up to 999 in Standard 3 and 9,999 by the end of Standard 4 (Malawi Institute of Education, 2013a, 2013b).

According to Wright, Stanger, Stafford, & Martland (2014) the traditional idea of teaching numbers from 0 to 9 for a long time, followed by the teaching of place value addition, started in the 1970s with influence from Piaget's (1953) theory of cognitive development. The idea was that the process of addition, regardless of the magnitude of the numbers being added, is based on the properties of the numbers 0 to 9. This approach was then supported by the development of base 10 materials during the same period. Wright et al. (2014), however, contend that contemporary teaching of number advocates introducing learners to numbers, even up to 100, as soon as possible. This would be followed by introducing them to informal

strategies for addition and subtraction, such as jump and split (see Figure 5-5 in section 5.5.1) before introducing them to the formal strategies and algorithms. They further argue that formal strategies (based on the place-value addition algorithm) tend to interfere with the informal strategies based on the development of number sense. The use of informal strategies for addition as well as the effect of formal addition algorithms have been discussed further under teachers' use of mediating talk and gesture for methods for generating solutions in section 5.5.1.

The role of the teachers' guide and learners' textbook

As stated in Chapter 4, the teachers mainly followed instructions provided in the teachers' guide when covering content within a particular unit (see Figure 4-47 and Figure 4-66). This is not unusual for primary school teachers in Malawi. A study involving some 14 experienced primary school teachers, confirmed that Malawian teachers tend to diligently follow the teachers' guide and learners' textbooks not as suggestions, but as prescriptions of what to teach and how it should be taught (Kazima et al., 2016). During the study, however, some of the teachers made slight adjustments to the suggestions given in the teachers' guide or the learners' textbooks.

Use of teachers' guide

During the first Standard 1 lesson on addition, the teacher made some adjustments to the instructions given in the teachers' guide shown in Figure 4-8. The instructions required the teacher to introduce the writing of an addition statement ($2 + 1 = 3$) by using a one-to-one correspondence of physical objects and number cards written on a piece of paper. Instead, the teacher opted to guide her learners through their thinking until they managed to come up with the same statement ($2 + 1 = 3$) on the chalkboard as shown in Figure 4-32. The teacher's engagement with the learners during the process opened up more learning opportunities. Findings from a study by Hoadley (2007) associated a teacher's control over the selection, sequencing and pacing of lesson content with higher learning gains. The Standard 4 teacher, on the other hand, closely followed the instructions in the teachers' guide on content selection yet maintained relative flexibility on the strategies for carrying out the tasks. The Standard 4 teacher also trained her learners to complete activities within the shortest time possible, rather than going by the pace of the slowest learners, which in turn negatively affects content coverage (Hoadley, 2012).

Use of learners' textbooks

The Standard 4 teacher followed the content in the learners' textbook as closely as possible. When giving learners work to do, she sometimes referred the learners to the textbook page where the problems were presented. In so doing, the teacher saved the time that would have been used for copying the problems on the chalkboard, and at the same time ensuring accuracy—considering that she worked with arrays of up to four 4-digit numbers.

However, there were two versions of the learners' textbooks: new and old version. Despite having the same set and order of problems on the page, the page numbers were different for the different versions. The Standard 4 teacher mentioned the page number for the version she had and did not have the page numbers for the other version at hand. The classroom confusion that resulted from the difference in page numbers could be a possible reason why the Standard 3 teacher may not have referred to the textbooks. During an interview, the Standard 3 teacher also commented on the need for taking note of the version of the learners' textbook that matched the teachers' guide. The age of the Standard 4 learners probably provided them with a relative advantage of following the teacher when navigating the textbooks as compared to the lower classes.

The Standard 1 teacher, on the other hand, opted to generate her own examples and not use the learners' textbook. She said that she seldomly used the learners' textbook to avoid issues such as illustrations that had the potential to confuse learners, as was the case with Figure 4-9 in section 4.3.2. It could be possible that since the range of numbers that were being used in Standard 1 were relatively small (0 to 5), the teacher found it easier to formulate the examples by herself.

During Lesson 4 of Standard 4, all the word problems examples were from the textbook and were all of the form: "Mwayi has X items, Chifundo has Y items, and Yamikani has Z items. How many items do they have altogether?" (see Table 4-22). These examples promote the 'combine and count all' strategy for addition. To promote the use of other strategies, one possibility was for the teacher to formulate word problems encouraging counting on, such as: "Mwayi had 8 sweets, his mother gave him 6 more sweets. How many sweets does he have?"

5.2.2 The modes of classroom interaction used during lessons

The tasks were presented and completed using various modes of interaction between the teacher and the learners (see Table 4-1). The graph in Figure 4-1 showed that, on average, whole-class teaching was the dominant form of interaction across the four classes. The learners in Standard 1 were organised in groups for most of the class time, probably as compliance to the emphasis placed on activity-based and learner-centred teaching approaches specified in the curriculum (Kazima et al., 2016; Malawi Institute of Education, 2012b). However, the nature of the activities that were being carried out in the groups seemed to facilitate the sharing of resources but not necessarily the sharing of ideas. Considering that the teacher/pupil ratio is the highest in the first two classes of primary school in Malawi (Ravishankar et al., 2016), it would not be unusual to find whole-class teaching in lower primary school classes. Whole-class teaching has often been criticised by advocates of learner-centred education, but it has remained the traditional form of classroom interaction in many developing educational systems (Mhlolo, 2013; Tabulawa, 2013). Askew (2019) points to evidence indicating that whole-class teaching can be effective. For instance, among the three Standard 2 lessons, the third lesson had the richest use of mediational means even though it was done through whole-class teaching with extensive involvement of individual learners.

Some teachers effectively used group work as one way of promoting learning. During Lesson 2 of Standard 4, for instance, the teacher applied some of the principles of the jig-saw technique (Aronson & Bridgeman, 1979) during group work. The teacher assigned unique problems written on chart papers to groups, and after all the groups had finished solving, the teacher swapped the groups' work such that each group had to study the work done by other groups and verify if they had solved their assigned problems correctly (see Figure 4-75). This approach saved the teacher's time, at the same time allowing multiple problems to be worked out and verified simultaneously. Thus, the teacher took advantage of the class size to do multiple

examples within the shortest time possible. This shows that the effectiveness of group work goes beyond putting learners in groups, but it depends on how the teacher designs the tasks. In their meta-analysis of international research on group work, Walshaw and Anthony (2008) found that group work was effective when the group size was relatively small (not exceeding five learners). In this study, however, the groups were large, having 10 to 15 learners per group. The teachers worked with large groups probably due to resource constraints. The Standard 2 teacher, for instance, opted to teach the whole of Lesson 3 without the use of group work (Table 4-14) when teaching addition using place-value boxes. During Lesson 3, the class had 94 learners present out of the 113 enrolled. If the teacher had opted to teach the lesson by dividing the class into, say, 10 groups (with 9 to 10 learners per group), she was expected to make 10 pairs of place-value boxes (that is, 20 boxes) for this lesson. An optimum group size of four to five learners would require 40 place-value boxes to be prepared before the lesson. In addition to the place-value boxes, the teacher would also be expected to prepare 50 sticks for each pair of place-value boxes, some of which would be in bundles of 10. This implies that 20 boxes would need 500 sticks to be prepared in advance for groups of nine to ten learners, while 1,000 sticks would need to be prepared for ideal group sizes with four to five learners. These demands could explain why the teacher might have opted to involve as many learners as possible during whole-class activities than to prepare resources for smaller groups. Another possibility was to ask the young learners to bring sticks and boxes with them when coming to the classroom, which would have presented the problem of variations in sizes of the items that could be brought by the learners and time to make the bundles and labels.

5.2.3 Lesson structure and sequence

Despite a few variations noted in some lessons, all the four teachers stated the same lesson structure and sequencing of tasks. The lessons started with an introduction that mostly involved a review of previous work followed by a whole-class discussion of an example given by the

teacher. This was followed by a task to be completed in groups. The lesson mostly ended with an individual task marked by the teacher. In some cases, the teacher gave homework when concluding the lesson. The observed consistent structure of mathematics lessons was also noted by Saka (2019). This harmony could be attributed to their use of suggestions from the teachers' guides, that give a general picture of how the lessons should flow. The teachers' guides offer suggestions on the activities to be done from the introduction to the conclusion of the lesson, as discussed in section 5.2.1.

It was noteworthy that even the Standard 3 teacher who taught mathematics only during this study followed the same structure. It might be possible that both the experienced and the new teachers in the study were following the structure they learnt during their teacher training, which includes all subject areas (Kazima et al., 2016). During an interview, the Standard 3 teacher in this study also recalled her last mathematics teaching experience while teaching Standard 2 during teaching practice. A study by Jakobsen et al. (2018), involving all the eight teacher training colleges in Malawi, indicated some improvement in the pre-service teachers' mathematical knowledge for teaching during their first two terms of study in college. The pre-service teachers' classroom teaching experience is probably enhanced by the supervised peer and microteaching they do in college as well as the one-year teaching practice at a primary school under experienced mentors (Kasoka et al., 2017).

5.2.4 Lesson duration

During the study, none of the lessons was taught within the official 30 minutes per lesson allocated for mathematics for Standards 1 and 2, or 35 minutes for Standards 3 and 4 (see Figure 4-1). Except for the first two lessons of Standard 1, the teachers seemed to have combined two single lessons for the days as allocated in official timetable to one double lesson. With this assumption, the average time of the lessons shown in Figure 4-1 indicates that the teachers for Standards 1 and 4 went beyond the duration of two lessons with over 10 minutes, while the

Standards 2 and 3 teachers taught below the time for two combined lessons by some 10 minutes. The possibility of combining lessons could partly explain the observation by Saka (2019) that the Standard 1 teachers in his study did not necessarily teach the number of lessons as suggested in the teachers' guide. Even though the durations of the lessons observed by Saka (2019) were not highlighted, he observed that teachers taught addition in only three or four lessons even if the teachers' guide proposed 12. For instance, the four observed Standard 4 lessons in this study were basically 8 lessons in terms of duration. Since the school allocated two short lessons but the teachers taught one long lesson, this might suggest that combined lessons were more convenient than split lessons for the large classes. The teachers ensured that the learners were still active by switching the forms of interaction during the long lessons to maintain extensive learner involvement. This could explain why the Standard 3 learners reported in section 4.5.2 showed signs of tiredness, such as yawning, during Lesson 4 of Standard 3, which was not the longest, but the same class resisted the teacher's closure of the longest Standard 3 lesson, that was completed in 1 hour 23 minutes. The learners still wanted to keep on working on the individual exercise that kept them engaged.

Teachers' strategies for reducing the duration of lessons

The teachers employed several strategies to ensure coverage of curriculum content while at the same time trying not to go far beyond the allocated time.

Reducing the number of problems given to learners during individual work

During interviews, the teachers stated that they gave careful consideration to the number of problems they gave to learners during individual work, mostly giving between two and three examples. As stated by the Standard 1 teacher, another strategy was limiting the number of days when homework was given to the learners, not exceeding two days per week, even though mathematics was taught every day. The teachers did this considering that the number of

problems to be marked is always a multiple of classroom attendance. The teachers' strategies were similar to the observations made by Graven (2016) during a study conducted in South Africa. She noted that mathematics teachers gave learners minimum written work just for the sake of policy compliance. Venkat and Askew (2018) also associated large teacher/pupil ratios with sporadic sets of learners' written work given by teachers coupled with the dominance of oral modes of teaching.

Doing corrections before completing marking

The teachers tried to reduce the overall time of a lesson by limiting the time spent on marking learners' work towards the end of the lesson. This was done by switching the order of the last two activities during each lesson: marking and working out the expected solutions with the class. Ideally, the teacher was supposed to finish marking and then invite the class to work out the given problems together. However, when working out the solutions, unmarked notebooks were withheld by the teacher. In Standard 3, learners were mandated to watch over their friends to ensure that they were not making the corrections in their notebooks, and learners were heard reporting those who were writing during the time for working out the solutions. The Standard 4 teacher also collected unmarked notebooks when she was about to work out the solutions for individual work and informed the class that she would not mark any work submitted after the problems were discussed. Those who failed were asked to rewrite and be marked the next day.

Since the goal of solving problems given as individual work with the whole class was to let learners make the necessary corrections, it is surprising that learners were not allowed to write during that session. It would appear as if the teachers offered the opportunity for doing corrections just as an obligatory requirement. However, it was noted that the intent of the teachers was not to let the learners to simply copy the corrections. During the session for working out the solutions with the class, those who had incorrect solutions were just given the opportunity to observe and follow the procedure carefully, and then use their effort to do the

corrections later. The teachers might have felt that the learning gains would be compromised if the learners were occupied with copying what was being written on the chalkboard.

Marking outside teaching time

As just discussed in the preceding sub-section, the teachers often reduced marking time during the lessons by collecting all the unmarked notebooks to be marked outside the teaching time. Though appearing as a solution, however, the Standard 4 teacher explained in an interview that it only worked when one had a partner teacher who would teach something else as the mathematics notebooks were being marked. Otherwise, the mathematics teacher would collect the notebooks, put them aside, and teach another subject. The teachers also made use of break time for marking, but the time was often not enough to finish all the notebooks, and it also meant that the teacher would also have no time to refresh.

Still, the Standard 1 teacher envied the senior classes for the ease to collect notebooks and mark later. In her case, collecting notebooks to be marked later posed additional challenges. As stated in section 4.3.2, the Standard 1 learners were yet to master writing their names on notebooks, and the notebooks would often have their covers detached as a result of lack of proper handling. Also, some learners brought notebooks of their siblings to class after losing theirs. Hence, after marking, the teacher would remain with some notebooks whose owners would not be easily identified, hence opted to finish marking during the lesson.

The Standard 2 teacher also faced the same type of problems observed in Standard 1. However, she solved the challenges by always asking the learners to write down their names at the bottom of the page where they had written their work, and thereafter collected the notebooks to be marked at another time. In an interview, the teacher also indicated that asking the learners to write their names every time gave them an opportunity to keep on practising writing, considering that mathematics should also complement the other subjects that teach writing

skills. As shown in Figure 4-1, the Standard 2 teacher generally taught her lessons in a relatively shorter time than the other three teachers.

5.3 Teachers' use of mediating artefacts

Physical artefacts are among the major cultural tools for mediating the learning of mathematics during the early years of primary school (Venkat & Askew, 2018).

5.3.1 The mediational role of artefacts

The teachers in this study worked with artefacts that were made by themselves or by the learners supported by their parents. The Standard 3 teacher, for instance, made several spike abaci at her home using clay in preparation for her lessons on addition with regrouping. In a study involving 14 Malawian teachers, they all agreed that “making teaching and learning aids” was part of the tasks of mathematics teaching (Kazima et al., 2016). The use of physical manipulatives makes learners see mathematics as a physical activity rather than a purely intellectual activity (Edwards, Moore-Russo, & Ferrara, 2014).

Even though the physical presence of artefacts during mathematics is essential, the MPM framework advocates teaching that moves toward the fading of artefacts from their physical forms to their ideal forms (Venkat & Askew, 2018). The fading of artefacts helps to develop learners' ability to work independently on cognitively demanding tasks in mathematics. The expectation is that the teacher should use the artefact not as an end in itself, but as a means towards learners' use of connected abstract mental structures. This is supported by the proposition that when humans are confronted with a novel situation where they are forced to use mental structures, they first rely on pre-loaded representations acquired through prior learning to work it out (Wilson, 2002). It has been theorised that even though the human body functions to distribute cognitive workload between mental and physical environmental structures, a more likely tendency is to offload the cognitive demand onto the physical

structures (Wilson, 2002; Wilson & Foglia, 2017) signified by the artefact. As noted by Askew (2019) when the teacher's mediation is "offloaded onto the artefact" (p. 218), the learners no longer see the need to look for mental conceptual structures that can enable them to make the required connections and generalisations.

5.3.2 Using artefacts for presenting tasks

Except for the Standard 3 teacher, the teachers used prewritten papers for presenting tasks, mostly those that were to be done in groups. The use of physical artefacts for presenting tasks was mostly observed in Standard 1.

The Standard 1 teacher worked with books, stones, leaves, and sticks for presenting tasks during the first two lessons. As stated in section 4.3.3, the teacher presented the tasks by making a verbal reference to the items that had been distributed to each group before the commencement of the lesson. For instance, during the first example in Lesson 1, finding $2 + 1$ was presented as "find two books plus one book." Thereafter, the teacher guided the learners through the process of adding the books and built on their thinking to write the resulting statement (see Figure 4-32).

The Standard 1 teacher demonstrated systematic fading of artefacts (Venkat & Askew, 2018) in her use of artefacts for presenting tasks within the first two lessons. During Lesson 1 tasks were solely presented using the physical objects that were given to groups. In Lesson 2, the fading of artefacts started with the representation of sticks using their corresponding drawings on the chalkboard (see Figure 4-18). By the end of the second lesson, the presentation of tasks switched to the use of drawings. The teacher also mediated addition using multiple artefacts and inscriptions for the same task. By varying the inscriptions and artefacts for the same task, the teacher made it possible for the learners to understand addition and see the connections between the activities and the written form in the textbooks.

5.3.3 Using artefacts for working out solutions to problems

The major use of artefacts by all the teachers was for working out the solutions to the given problems. As presented in Chapter 4, the teachers worked with loose counters, framed counters, place-value boxes and abaci.

Use of loose counters for solving problems

After using loose counters for presenting tasks in Standard 1, the teacher also used them for working out the solutions to the presented problems. By using different types of artefacts (books, stones, leaves, sticks), the teacher made it possible for learners to discern that the process of addition is independent of the nature of objects being counted. Thus, the Standard 1 learners could perform the addition of any given numbers using any available artefact. The Standard 4 teacher also used loose counters in a plate for working out solutions during the first two lessons. Since the loose counters were not organised in any mathematically intuitive way, they could be classified as *unstructured artefacts* in the MPM framework (Venkat & Askew, 2018).

When working with the loose counters, the sum was found by representing each number with the corresponding number of counters followed by counting them all. By counting all, the structural relationships of numbers were concealed and the efficiency of working with counters was reduced. The MPM framework refers to this as unstructured use of artefacts (Venkat & Askew, 2018). This usage of physical manipulatives was also observed by Saka and Roberts (2018) in Standard 1 Malawian classrooms. The potential for the use of counters during addition could be enhanced by counting on from the first addend to the next.

Use of framed counters for solving problems

Even though the teachers worked with various types of artefacts, the hand-made framed counters appeared to be the main mediational tool used by both the teachers and the learners

in this study. The framed counters—called “the Malawian bow-abacus” by Saka and Roberts (2018, p. 391)—has been part of the Malawian mathematics classroom culture for generations. Contrary to the findings from this study, however, Saka and Roberts (2018) reported that they did not observe teachers using the framed counters in the five Standard 1 classrooms they studied. They said that they only observed learners using it on their own when carrying out addition.

During the counting process, the teachers pushed the counters one-by-one, corresponding to each number (see Excerpt 4-14). Even when working with the addition of zero, the teachers represented adding zero by just sliding fingers along an empty section of the string of the framed counters. This process was useful in Standard 1 in that it helped the young learners relate the number words verbally mentioned to their corresponding physical quantity of counters the teacher was pushing to one side. The constraints of unit counting when using artefacts have been discussed the teacher further under “mediating talk and gesture for providing methods for generating solutions” in section 5.5.1.

Use of fingers for solving problems

The use of fingers maintained a secondary role during the lessons in the study. Learners were only asked to use fingers if they had not brought their counters. While working out sums between 10 and 20 learners in Standard 2 were asked to include their toes in the calculations (see Utterance 281 of Excerpt 4-14). As mentioned in Chapter 1, Brombacher (2011) showed the need for familiarising learners with the use of fingers after more than half of Standard 2 Malawian learners who participated in his study ($n = 500$) failed to add single-digit numbers with sums less than 10. Wright and Ellemor-Collins (2018) recommended that the use of fingers be limited to sums in the range of 1 to 10, and prioritise informal strategies for addition and subtraction from 11 to 20 (discussed in section 5.5.1). With this approach, the use of toes might seldom be required when carrying out addition.

5.3.4 Using artefacts to promote learner engagement

The teachers also used artefacts for class management, ensuring that every learner was participating during the lessons, including slow learners.

Use of framed counters for learner engagement

The teachers also relied on framed counters as a cultural tool for promoting active learner engagement for their large classes. When the teachers pushed the counters, they encouraged each learner to be doing the same, especially in Standards 1 and 2. The requirement that each learner had to use personal counters ensured that everybody's mind focused on the activity at hand. As stated at the beginning of this section (5.3), the learners' physical involvement with the counters enabled them not to view mathematics as a purely intellectual activity (Edwards et al., 2014). The Standard 4 teacher confirmed this point by saying: "If we just explain to a child without using any object, it is sometimes difficult for them to understand clearly. But when they use a real object, they can even practise using it when alone...."

When introducing Standard 1 learners to the use of the framed counters, the teacher taught them how to hold it; raising it close to the forehead with one hand, leaving the other hand free to push the counters (see Figure 4-12). The same posture was also noted with the Standard 4 teacher (see Figure 4-83). When learners held the counters using the posture demonstrated by the teacher, it made it easier for the teachers to quickly note the non-participating learners. In all the Standard 1 lessons, the teacher was observed interrupting the counting after noting non-participating learners. Even during group activities, the teacher observed if everyone was following the counting. This can be contrasted with the difficulty that the teachers faced to notice those who were not writing during individual work, and had to be reported to the teacher by other learners. Even when the one who was not writing was reported to the teacher, the teacher kept on asking a number of times to identify the location where the learner was sitting.

Using fingers for learner engagement

The teachers asked learners who did not have counters to use their fingers. One of the main reasons for asking learners to use fingers was that the learners should not just be idle, but actively participate during the lesson. It was noted during the lesson that when learners were using fingers, they raised their hands for the teacher to notice their participation (see Figure 4-15 and Figure 4-16). It could be assumed that the learners had been trained to raise their hands when counting with fingers for the teacher to see quickly the ones who were actively participating.

5.3.5 Using artefacts for modelling the process of addition

Niss (2012) points to modelling as the major reason that makes mathematics to be the single largest educational subject in the world. Teachers of learners in the early years of primary school often use models to make abstract mathematical concepts become meaningful to them.

The role of artefacts for modelling mathematical concepts

There are some contrasting perspectives in the use of artefact-based models in mathematics teaching, such as “modelling for the learning of mathematics” and “learning mathematics for modelling” (Erbaş et al., 2014, p. 1622). The “modelling for the learning of mathematics” perspective uses modelling to enable learners to learn fundamental mathematical concepts, thus, moving from concrete models to abstract concepts. This perspective is based on the assumption that learning is meaningful through the use of models, as partly discussed in section 5.3.1. The “learning mathematics for modelling” perspective is aimed at generating models from mathematical concepts, thus moving from abstract to concrete. Stohlmann and Albarracín (2016) cite research evidence on early grade learners’ capability to model different situations.

Instead of dwelling on which of the above two perspectives is ideal, Saka and Roberts (2018) as well as Venkat and Askew (2018) recommended moving between representations when

using artefacts in an early grade classroom. This makes the two perspectives complementary. As such, it can be left to the discretion of the teacher to decide which aspect of a particular artefact might need to be directed to the learners' attention in order to bring up the desired mathematical meaning.

The Standard 2 teachers' use of place-value boxes for modelling addition

The Standard 2 Mathematics Teachers' Guide refers to the use of place-value boxes as tools for modelling addition (Malawi Institute of Education, 2012c). Probably, the idea was that once the learners had grasped the underlying concept of place-value addition, they could proceed with the use of counters.

The Standard 2 teacher appeared to have opted for moving between representations (Saka & Roberts, 2018; Venkat & Askew, 2018) rather than going by the instructions numbered 6, 7, and 8 shown in Figure 4-47 from the teachers' guide. The instructions required the teacher to use counters independently from the use of place-value boxes, but the teacher used both artefacts concurrently. This made the use of place-value boxes during the lesson to be temporary because the teacher and learners also used counters to solve the same problems. This concurrent usage of place-value boxes and counters made it possible for the teacher to fade away (Venkat & Askew, 2018) the place value boxes after the underlying concept was fully grasped by the learners. This was noted during individual work where the learners only worked with counters.

However, when working with place-value boxes, the Standard 2 teacher started counting the bundles followed by the single sticks. This meant that the sum was found by first adding tens followed by ones. This worked because all the examples did not require regrouping. It was a requirement that the numbers that are given at this stage be chosen in such a way that the sum of digits in each column does not exceed 10; hence, teachers could "consider any two numbers

that involve addition without regrouping” (Malawi Institute of Education, 2012c, p. 28). This made the procedure of starting with bundles (tens) followed by ones localised for such examples. Although the learners would not see any effect on the answers found regardless of where they start from at the moment, starting with adding ones would be the best practice when they start handling the addition of numbers requiring regrouping in upper classes. This could be the reason why the teachers’ guide reminds teachers to emphasize to learners that “addition should begin with the ones then the tens” (Malawi Institute of Education, 2012c, p. 29).

The Standard 2 teacher used the count-all strategy of addition when working with bundles and single sticks in place-value boxes as well as counters. The use of place-value-boxes could have provided other opportunities for learning because of their nature. For example, the sum was not represented by a third box. The teacher added the contents in the box for the second addend to the box for the first addend, making it ideal for the count-on addition strategy. It would have been more efficient to count-on from the bundles and sticks already placed in the place value box for the first addend. The opportunity to count on was missed because the teacher mixed the contents of the two place value boxes then counted all.

The Standard 3 teachers’ use of spike abaci for modelling addition

The Standard 3 learners were already familiar with formal methods for adding multi-digit numbers, but they were taught how to model the same process using a spike abacus. It can be implied from the Standard 3 teacher’s statement in Excerpt 4-27, that her objective shifted from teaching the process of addition to teaching the representation of addition using the abacus. Thus, the teacher helped the learners to understand the abacus as a mathematical object in itself (Sfard, 2008). Due to this goal, the Standard 3 teacher had to re-direct learners to the abacus when she noted those who quickly turned to pen and paper as the teacher was waiting for them to use the abaci. This enabled the learners to be familiarized with the spike abacus as one of

the cultural tools (Wertsch, 2017) for modelling addition involving regrouping. As stated by the Standard 3 teacher, the use of abaci modelled well the concept of place-value and carrying-over to the next digit (regrouping), which would not be possible with the use of the traditional framed counters. During an interview, the Standard 4 teacher also commented that she only used the abacus when working with topics that directly focused on learning the abacus. This might also imply that the abacus took the role of a mathematical object whose use had to be taught separately.

5.3.6 Using artefacts for showing mathematical connections

The Standard 2 teacher used prewritten papers to show mathematical connections across examples (see Figure 4-48). Her aim was to lead the learners to number bonds or “pair-wise configurations” (Wright & Ellemor-Collins, 2018, p. 20) of 10 and 12. This was achieved by displaying all the papers at the same time by learners standing in the front of the classroom (see Figure 4-58). The way the four teachers achieved mathematical connections will be discussed under mediating talk for building mathematical connections in section 5.5.2.

5.3.7 Using artefacts for enhancing content coverage

Using the prewritten papers enabled the teacher to cover more examples in class. This is because it saved time than writing the examples on the chalkboard and letting learners copy from the chalkboard. For instance, the Standard 1 teacher worked with more examples in Episode 3 of Lessons 4 to 6 where she used prewritten papers compared to the other episodes where the examples were written on the chalkboard (see Table 4-9). Likewise, the Standard 4 teacher worked with more examples during Episode 3 of Lesson 3 and Episode 2 of Lesson 4 due to the use of prewritten papers compared to the other episodes within the same lessons where she wrote the examples on the chalkboard (see Table 4-21).

During an interview, the Standard 4 teacher also explained that she used chart-sized papers pasted on the walls to let learners refer to them during their free time after the lesson, such as break time. Just like the Standard 4 teacher, the Standard 1 teacher also pasted the papers with examples solved by learners in groups on the walls. This technique was possibly useful for enabling learners who might have been absent to follow the lessons they missed. Thus, it can be said that the use of prewritten papers reduced lesson time while extending teaching time. These charts also provided an opportunity for the teachers to make vertical connections of examples discussed during the lesson with those that were done during the previous lessons.

5.3.8 The mediational potential and limitations of framed counters and spike abaci

As discussed in the previous sections, framed counters took a central role in the teaching of mathematics across all the classes. The focus of this sub-section is to discuss the merits and demerits of the structural properties of this cultural tool as well as the observed usage patterns and suggest possible improvements. The section also discusses the limitations in the teachers' use of spike abaci followed by suggestions on how they can be used to show properties of numbers.

The potential use of framed counters

The framed counters were the most readily available artefacts in the classroom because they could easily be made by the learners at home. Being carried by the learners to school every day, the framed counters assumed an integral part of the learners' mathematics life. This could explain the Standard 2 teacher's seeming use of counters as the de-facto standard for assessing the accuracy of place-value boxes during Lesson 3. During Lesson 1, the Standard 3 teacher also used framed counters for verifying the solutions found during individual work, yet the focus of the lesson was on the use of abaci for making calculations. The use of framed counters also saved the teacher's time for lesson preparation. The Standard 1 teacher explained during

an interview that the use of framed counters saved the time that would be spent on gathering single-use counters (such as leaves and sticks) from the school environment.

Limitations in the use of framed counters for promoting mathematical cognition

Despite their wide popularity, the framed counters had some limitations associated with their physical structure as well as their use in the classroom. For instance, the framed counters were not standardised. Learners were seen carrying different sets of counters— some had as few as 11 counters as in Figure 4-3(a), while others had as many as 62 counters as in Figure 4-3(b). During a post-lesson interview with the Standard 2 teacher, she indicated that she made a sample of framed counters and asked the learners to make something similar at home and thereafter carry it with them to school. However, she indicated that the number of counters to be fitted on each frame was not specified to the learners. For learners who were learning addition of numbers with sums not exceeding 20, the 11 counters were inadequate while the 62 counters were too many. A total of 20 counters would have been ideal, and this might have been achieved if the instructions to the learners specified the number of counters.

It can also be noted that the counters were not arranged in a pattern that could intuitively show number structures that could promote subitizing. Subitizing is understood as “the quick and accurate enumeration of small quantities” (Katzin, Cohen, & Henik, 2019, p. 790). Since the basic mechanism of subitizing is the recognition of patterns, Katzin et al. (2019) argue that any quantity can be subitised as long as it constitutes a recognizable pattern. Due to the absence of a structured pattern, the counters promoted unit counting. This made the framed counters be classified as unstructured artefacts using the MPM framework (Venkat & Askew, 2018).

Possible use of framed counters to promote mathematical cognition

Saka and Roberts (2018) echoed the point made by Askew (2012) that the effective use of a representation is a process that improves with time to reach a point where the representation

can be considered a mathematical model. As such, despite the observed unstructured nature of framed counters discussed in this study, some improvements can be made to transform them into structured mathematical models that can be used for modelling the properties of numbers, like an abacus.

Restructuring the arrangement of counters on the frame

One way of using artefacts to enable the development of mathematical skills such as subitising is to adopt the notion of embodied cognition, which posits that the properties of an agent's body constrain the concepts and representations it can process by its cognitive system (Shapiro, 2019; Wilson & Foglia, 2017). This implies that the understanding of concepts in one's surrounding world depends on the nature of our bodies. As mentioned towards the end of section 5.3.4, a direct application of this theory involves the use of fingers for counting as well as performing addition and subtraction from 1 to 10, followed by informal strategies for addition and subtraction from 11 to 20 (Wright & Ellemor-Collins, 2018). Another application of embodied cognition might require re-structuring the bow-shaped framed counters shown in Figure 4-2 to mimic the idea that our bodies have five fingers in each hand (totalling 10), as proposed by Saka and Roberts (2018). This can be done by arranging the counters in groups of five or 10, using alternating colours, shapes, or sizes that display recognisable patterns.

During the study, some learners were observed using framed counters with more than one type of counters, which just required slight modifications to bring out patterns (see Figure 5-1).

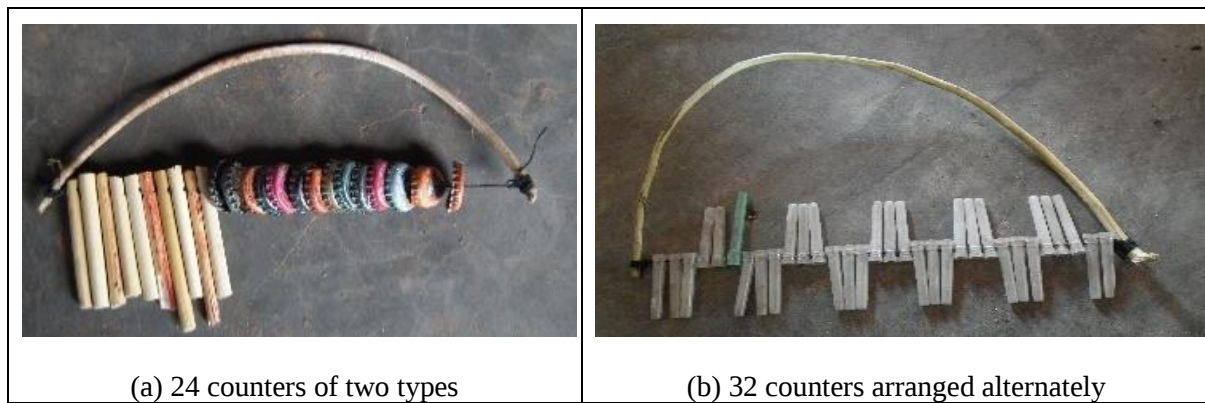


Figure 5-1: Some learners' framed counters of different types and arrangement (Source: Researcher).

The framed counters shown in Figure 5-1(a) had 10 straws and 14 bottle tops. These would just be arranged in four alternating groups of five straws and five bottle-tops to make a standard set of 20 counters as shown in Figure 5-2(a). The learner in Figure 5-1(b) arranged his counters in a temporary alternating pattern while playing with his counters on the floor during idle time. The pattern made by the learner in Figure 5-1(b) had groups of three counters, and could be used for counting in threes or it could easily be extended to groups of five, for counting in fives.

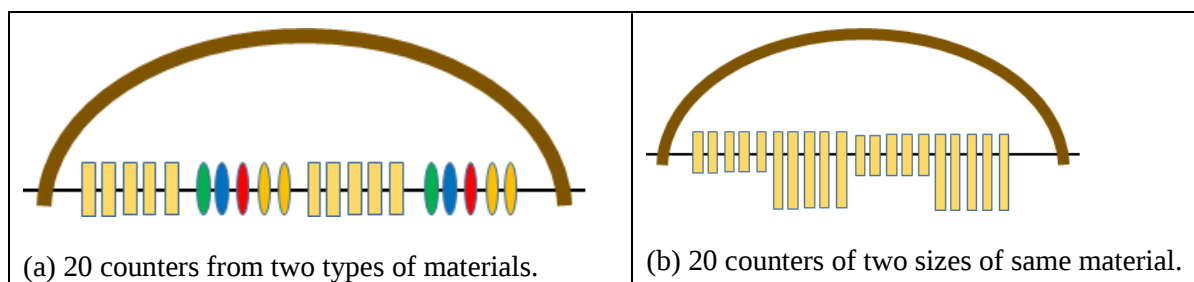


Figure 5-2: Standardised bow abaci with 20 counters (Source: Researcher).

The models shown in Figure 5-2 may not require sorting the counters according to colours as highlighted in a similar model by Saka and Roberts (2018). The use of different types or sizes of materials may be adequate to highlight the desired structure that can promote subitizing.

Specifying the number of counters on a frame

To avoid the cases shown in Figure 4-3(a) and Figure 4-3(b), Saka and Roberts (2018) recommended limiting the number of counters on each frame based on the range specified by the curriculum. As such, the number of counters on the frame may be incremented gradually from an upper limit of 5 followed by 9 in Standard 1, then extending to 20 in Standard 2. Even though the number range extends to 99 in Standard 2, the idea of using unit counting when performing addition and subtraction may need to have been faded out (Venkat & Askew, 2018) by the time they reach an upper limit of 20. As such, it would not be necessary for a learner to carry the 62 or 32 counters shown in Figure 4-3(b) and Figure 5-1(b) or any number of counters more than 20.

The usage limitations of spike abaci

When performing addition, the Standard 3 teacher devised her own strategy of raising one counter to represent a group of ten counters from one spike to be moved to the next place value. The teacher had to find her own representation that would convince her learners because she could not find an explanation on how this could be done in the teachers' guide. The challenge was not unique to the teacher because the spike abacus is ideal for modelling the positional notation of numbers. As such, it only displays counters that directly correspond to written numbers and, conversely, hides what is hidden in the positional notation of numbers (Speiser & Walter, 2011). When using a spike abacus, a second level of mediation is required to show structural properties of numbers, such as showing that ten counters on one spike are equivalent to one counter on the next spike to the left. The second level of mediation could have been achieved using strategies discussed in the next subsection.

Possible use of spike abaci to promote mathematical cognition

The abaci used by the Standard 3 teacher (see Figure 4-67) were able to represent the addends and their sum with no need of further mediation when the sum of counters along two corresponding spikes was less than 10, as was the case with the examples used during the first two lessons (see Table 4-20). During the first lesson, the teacher enhanced the mediation of relative positions of the place values by specifying the colours of counters (bottle-tops) to be placed along each spike as illustrated in Figure 5-3 that follows.

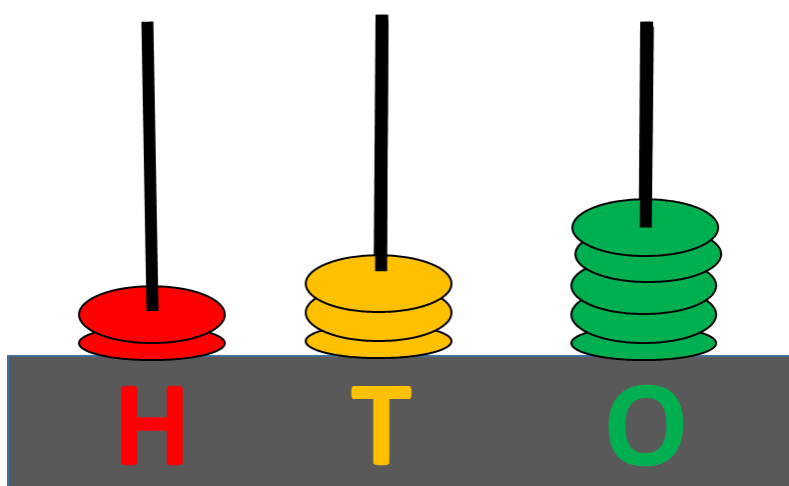


Figure 5-3: An illustration of colour coded representation of 235 using counters on a spike abacus (Source: Researcher).

With the representation shown in Figure 5-3, the colours provided a second check for errors that resulted from the inadvertent flipping of abaci, as noted during the lessons where colours were not used.

The visual connection that the teacher made by raising one counter to represent ten counters of the same type could have been enhanced if she had maintained the colour coding of place values. In that case, ten counters of one colour from a spike would mean one counter of another colour from the adjacent spike to its left and vice-versa. When carrying out the addition of numbers represented by two abaci, then ten counters of one type would be exchanged with one

counter of the next value (Bussi & Mariotti, 2008). The use of colour coded counters can be considered a feasible option because coloured bottle-tops were easily available within the communities surrounding the school.

Alternatively, the teacher could also work with counters made from straws of different types to signify the magnitude of the place values, as shown in Figure 5-4. In that case, the representation of 10 smaller counters with one relatively larger counter would signify the notion of magnitude of the place-values to the learners.

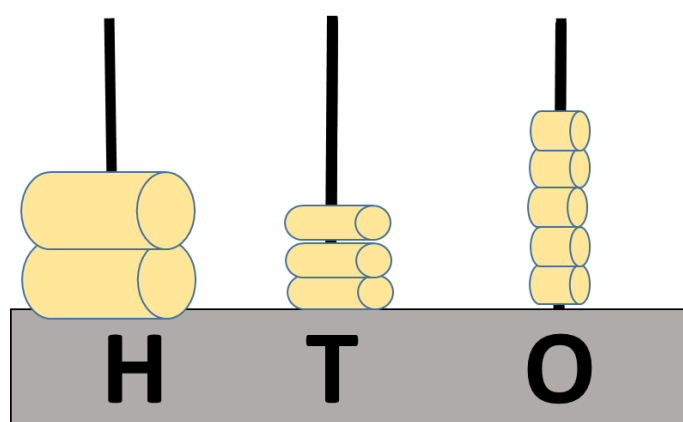


Figure 5-4: An illustration of the representation of 235 on a spike abacus using pieces of straw (Source: Researcher).

As shown in Figure 5-4, the size of the two counters would intuitively convince learners that they represent 200 as compared to only giving a verbal explanation.

5.4 Teachers' use of mediating inscriptions

The teachers generated various types of inscriptions on the chalkboard during teaching (Venkat & Askew, 2018). The teachers mainly used the chalkboard inscriptions for the presentation of tasks and examples, the reification of mathematical objects, as well as showing the method used for generating solutions.

5.4.1 Use of inscriptions for the presentation of tasks and examples

The teachers mostly used the chalkboard for presenting the examples to be worked on (see Figure 4-19, Figure 4-53, and Figure 4-4). The examples were mostly structured addition statements, except for the first three Standard 1 lessons where the teacher used drawings of physical objects (Figure 4-19).

The Standard 2 teacher achieved a certain measure of structure in her presentation of examples. In Episodes 2 and 3 of Lesson 1, the teacher presented the examples in pairs; $1 + 9$ was presented with $6 + 4$ while $2 + 8$ was presented with $5 + 5$ (see Figure 4-53). This made it easy for learners to see that in each case, the addends had a sum of 10. This was necessary for learners to make a comparison at a glance rather than reverting to their memory from a previously worked out problem if the examples had been worked out separately and erased from the chalkboard. The structuring would have been strengthened further by proper sequencing of the same examples to show number properties (such as presenting $1 + 9$ with $2 + 8$, and $5 + 5$ with $6 + 4$). This would enable the learners to notice some variant and invariant aspects of the example spaces (Goldsmith & Seago, 2011; Kullberg et al., 2017; Mhlolo, 2013; Schifter, 2011). The affordances of using a structured presentation of examples have been further discussed under the teachers' mediating talk and gesture for building mathematical connections in section 5.5.2.

5.4.2 Use of inscriptions for reifying mathematical objects and processes

The ideal use of inscriptions enhances the seamless transition from the presence of physical manipulatives to numeric abstraction (Venkat & Askew, 2018). This movement between physical and written representations was demonstrated by the teachers for Standard 1 (see Figure 4-18) as well as Standard 2 (see Figure 4-54).

The Standard 1 teacher used multiple inscriptions in varying levels of structure: starting with unstructured drawings of objects, followed by horizontal and vertical mathematical statements—thus reifying the process of addition (Sfard, 2008). The Standard 1 learners were trained to flexibly move between representations by requiring them to present their solutions with numbers written below the corresponding drawings. In Standard 2, the seamless movement between physical and written representations was mostly noted during lesson 3. The teacher placed the physical representation of an addend in a place-value box below its written representation (see Figure 4-54). After finishing solving the problem using the place-value boxes, the teacher wrote the problems again as structured mathematical inscriptions (see Figure 4-55), thereby reifying (Sfard, 2008) the physical and verbal representations of the addition process into its corresponding mathematical objects (structured addition inscriptions).

5.4.3 Use of inscriptions for recording methods for generating solutions

Starting from Standard 1, the learners were taught to show how they arrived at the solution. The first use of inscriptions for working out problems involved counting drawings to find the answer (see Figure 4-18 and Figure 4-21). The answer was given by drawing all the given representations to the right of the equal sign. The teacher repeatedly reminded learners on the need to present answers in the same form as the question was presented (such as to “draw all the trees”). However, as explained in section 4.3.4, most learners kept on just writing the numerical value of the answer without showing the required method (see Figure 4-20 and Figure 4-22). By turning back learners who had given the correct answer without the corresponding drawings, the teacher made it possible for the learners to see that the method used to obtain the answer is important in mathematics. The drawings were used to provide the justification for the correctness and incorrectness of the learners’ solutions, as specified under the teacher’s mediating talk for advancing learning connections in the MPM framework (Venkat & Askew, 2018). The teachers for Standards 3 and 4 used arrows and inscriptions

written under the bottom horizontal bar of the given problem as a way of recording the steps followed when carrying out the regrouping algorithm (see Figure 4-73 and Figure 4-85).

5.4.4 Use of inscriptions for promoting learner engagement

All the teachers actively involved learners in reading and making chalkboard inscriptions. After writing the example on the chalkboard, learners were invited to read the entire statement, or they would read parts of it in turns. When working out the answer, learners were also asked to come and write the required inscriptions at each stage. The Standard 3 teacher demonstrated some gender sensitivity through statements like: “The second one should be worked out by a boy” , to encourage participation of both girls and boys.

By asking learners to take turns in reading and writing numbers on the chalkboard, they were given more opportunities to master the new numbers, considering the spiral nature of the mathematics curriculum discussed in section 5.2.1. As shown in Figure 4-63, some Standard 2 learners appeared not to have yet mastered how to write the numbers 16, 18 and 19. The common error was to switch the order of the digits and write 61, 81 and 91 respectively. Richmond and Taylor (2014) explained that this error is due to children’s visual perception. Some Standard 3 learners also had difficulties in reading some of the three-digit addends presented on the chalkboard. As observed by Richmond and Taylor (2014), the underlying causes of errors associated with visual perception are not self-correcting with age, hence require explicit remediation by the teacher. The teachers achieved this by always asking the learners to read the numbers presented on the chalkboard and letting them write the solutions. Thus, by making the learners read and write chalkboard inscriptions, the teachers promoted the learners’ engagement and their learning.

5.5 Mediating talk and gesture

This section discusses three types of teachers' mediating talk and gesture as specified by the MPM framework. This includes mediating talk and gesture for providing methods for generating solutions, building mathematical connections, as well as advancing learning connections.

5.5.1 Mediating talk and gesture for providing methods for generating solutions

The main strategy for addition used by all the teachers was counting all. This strategy was used both when working with single-digit addends in Standard 1 and when working with the multi-digit place-value algorithm in Standard 4.

Finding the sum by counting all

As discussed in section 5.3.3, the primary use of artefacts was for working out solutions to problems, and this was mostly done using the count-all strategy. This approach promoted learner engagement, which in turn, made the learners view mathematics, not as a solely intellectual activity, but a physical experience (Edwards et al., 2014). With the count-all strategy, the lessons progressed at a pace that enabled even the slowest learner to follow the method used.

Despite the possible benefits that the count-all strategy might offer, some reasons might not make the method beneficial in the long run. The limitations of the count-all strategy include increased probability of propagating errors and misconceptions, delaying content coverage, and deterring learners' metarepresentational competence.

Increased likelihood of propagating errors

The probability of making errors when generating solutions with the count-all method is heightened by the unstructured nature of the counters used by the teachers. For smaller sums

of less than 10, learners could count the counters for each addend and the resulting sum with lesser chances of counting errors. However, when the sum of the addends approached 20, the method exposed the learners to more possibilities of making errors. For instance, when finding $14 + 5$, the learners in Figure 4-61 missed the correct answer with a ± 1 error, finding 18 or 20. It can be assumed that the ± 1 error came about due to the unit counting. The two possible sources of the ± 1 error could be when counting 14, or when counting the resulting sum, 19. If the learners had used the count-on strategy, they would have just counted five more from 14 to reach the correct answer, 19, rather than starting again from 1. The count-on method could also be enhanced with structured counters as proposed in Figure 5-2, where 15 would quickly be counted in fives by pushing three groups of 5s.

Increased likelihood of propagating misconceptions

The use of the count-all strategy for addition also posed possibilities of propagating misconceptions. This challenge was noted when one of the addends was 0. When working with 0 as an addend, some of the teachers also expected the class to count the 0 and add it to the other addend. For instance, when working out $2 + 0$ with the class during Lesson 4 of Standard 1, the teacher remained consistent with the requirement for counting each addend (see Utterance 303 in Excerpt 4-10). So, the Standard 1 teacher counted 0 by sliding her fingers on an empty string. This appeared to be the teacher's physical representation of 0. The same approach was also used by the Standard 4 teacher (see Figure 4-83). The challenge, however, is that by their nature, physical models are open to different conceptions, hence proper meaning-making largely depends on the teacher (Saka & Roberts, 2018). To minimise unintentional propagation of misconceptions with a particular physical model, some of the teachers often used multiple representations of the same concept using other means of mediation, such as the Standard 2 teachers' gestures for representing 0 (see Figure 4-30 and Figure 4-59). The Standard 2 and Standard 3 teachers also added 0 based on the premises that

“0 means nothing” or “adding 0 is the same as not adding” (see Utterance 261 in Excerpt 4-18 by the Standard 2 teacher). Here there was also need for the teacher’s talk to be carefully worded so as not to lead learners to perceive zero as not a number, because the misconception that zero is not a number can arise unexpectedly in the classroom (Muir, 2008). These challenges that required proper use of talk to avoid propagating errors arose because of the teachers’ consistent requirement for unit counting when generating solutions to problems.

Delaying content coverage

The count-all strategy normally makes every example be treated from scratch, making it a localised method for generating solutions to problems (Venkat & Askew, 2018). The teachers’ requirement for the class to perform unit counting, even after the class had mentally given the answer, was time consuming. This possibly contributed to the long durations of the lessons as shown in Figure 4-1. Though having the strength of including even the slowest learners in the process of working out solutions to problems, Hoadley (2012) commented on the resulting negative impact of such slow lessons on content coverage.

Deterring learners’ representational competence.

Starting from Standard 1, learners appeared to have some potential to do more than what they were formally expected to do, but the teachers sometimes rejected their quick offers. In Excerpt 4-9, for instance, a learner intuitively gave an answer before the class had physically counted, and the teacher reprimanded the learner saying: “How have you known that it is 4? So, I want you to pick them one by one and count with your friends. Alright?” In Utterance 38 of Excerpt 4-19, the Standard 2 learner (Learner 8) who raised a hand answered “12” at a time when the teacher was expecting the learners just to read the given statement ($3 + 9$). After the discussion shown in Excerpt 4-19, the teacher continued with the formal procedure for adding the two numbers using counters, as shown in Excerpt 4-20. As seen in Excerpt 4-20, the answer was

formally found by the teacher and the class in Utterance 57, yet Learner 8 had found it earlier in Utterance 38 of Excerpt 4-19. Similarly, during Lesson 2 of Standard 4, it was surprising that the teacher insisted on unit counting for $11 + 1$ even though the learners had already given 12 as the answer. Adding to the surprise was the observation that during Lesson 1, the class had been asked to mentally work out sums of numbers that were much larger than $11 + 1$. During an interview, the teacher mentioned accuracy as the main reason for requiring unit counting from the learners. However, during learner-led class discussions, the learners in front worked out the problems without using any counters.

When a learner was coming to the front to solve problems without carrying anything, the Standard 2 and Standard 3 teachers would ask them why they were not carrying counters, and would then ask the classmates to lend their counters to the learner. By asking them about where their counters were, the teacher was insisting that the learners use counters. It could be assumed that the teacher thought that learners would not work out the problems correctly without counters, yet the learners had already demonstrated that they could work out the additions without counters. Such incidents signified “teacher talk that ‘pulls back’ towards more naïve strategies than those offered by students” (Venkat & Askew, 2018, p. 79). The Standard 2 teacher explained that learners working out problems in the front had to use counters because they assumed the role of the teacher (who always needs to use teaching aids). Unlike the teachers of Standards 2 and 3, the Standard 4 teacher did not ask the learners going to the front to carry counters with them.

With routine use of unit counting, development of learners’ abilities for making their conceptual representations may be negatively affected. This might be so because when presented with a cognitively challenging demand, just like any human, the learners start working it out by first using their pre-loaded mental representations acquired through prior learning (Wilson, 2002). Stohlmann and Albarracín (2016) and diSessa (2004) cited research

evidence that supports learners ability to work with different representations including algebra even before they are introduced to formal methods in the classroom. It is after failing to solve it mentally that they may resort to transfer the cognitive demand to a readily available artefact (Askew, 2019; M. Wilson, 2002; R. Wilson A. & Foglia, 2017). Oftentimes, learners who rely on routing counting may not be able to recognise incorrectness of the solution they have found based on the aspects such as the magnitude of the numbers they are working with (Aploon-Zokufa, 2013; Graven, 2016). Observations from studies conducted in South Africa indicated cases of young learners who could perform complex arithmetic algorithms but failed simple problems that required understanding of number sense (Askew, 2013; Graven et al., 2013).

Perfunctory counting

During the study, Standard 3 learners showed some signs of their reaction to the requirement for concrete unit counting by doing it in a perfunctory manner. This might have resulted from instances like Excerpt 4-28, where the whole class quickly gave 8 as the answer for $2 + 6$ but the teacher said: “No! You should count what you have in the abacus.” Since the class already knew the outcome of the counting, they counted in a perfunctory manner that indicated that they were doing it just to fulfil the teacher’s requirement. The perfunctory counting was more pronounced when it was being led by a fellow learner working out a problem in front of the classroom. The class counted faster than the one who was picking the counters despite being reminded by the teacher to wait for the one physically handling the counters. The class would quickly rush through the counting and reach, say 8, while a fourth or fifth counter was just being picked in front.

Mental methods

The teachers for Standards 1 and 2 gave learners an opportunity to use mental methods. During Lesson 2 of Standard 1, the teacher gave two problems for the learners to work out mentally

and just give the answer. Mental strategies were useful when verifying the correctness of their solutions. For instance, Standard 1 learners could not tell that the answer offered by their classmates was wrong until they later checked with the teacher using counters, even when one addend was zero. This could signify that the learners may not have developed mental calculation strategies for working with the addition of zero, or they only expected to do it first with artefacts before they would be sure of the correct answer. The Standard 4 teacher also asked the class to solve some six addition problems mentally during the introduction and the conclusion of the first lesson. The teacher seemed to follow the Standard 4 teachers' guide, which included some 2-digit problems for learners to work out mentally at the beginning of the lessons (Malawi Institute of Education, 2013b).

Learners' flexibility in carrying out addition in the absence of artefacts can be developed using well-planned mental addition practices (Wright & Ellemor-Collins, 2018). Evidence shows that informal mental methods of calculation may not necessarily be guesswork or recall of facts, but are often linked to a deeper understanding of higher-order number concepts (Ruiz & Balbi, 2019). Rather than only focusing on the 2-digit addends suggested in the teachers' guide—for Standard 4 learners who were doing 4-digit arithmetic—mental methods would further be enhanced by carefully selecting larger sets of problems followed by asking the learners to explain how they worked it out. Anthony and Walshaw (2009) recommended mentally challenging tasks that would provide learners with opportunities to struggle with ideas with increasing levels of sophistication, rather than just focusing on obtaining the correct answer. A possible example can be $5678 + 999$ where learners can work out it out using mental strategies such as compensation (Parker & Faulkner, 2004), that is, $5678 + 1000 - 1$.

According to Wright et al. (2014), most of the mental strategies used in additive relations can broadly be grouped as either jump or split strategies. Figure 5-5 illustrates 2-digit addition

using the jump-strategy on an empty number line. The example $(35 + 13)$ was solved using place-value boxes during Lesson 3 of Standard 2.

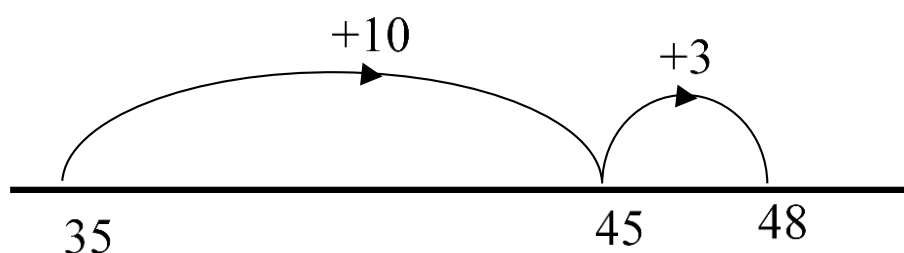


Figure 5-5: Illustrating $35 + 13$ using the jump strategy on an empty number line (Source: Researcher).

The example in Figure 5-5 could also be solved using the split strategy as $30 + 10 + 5 + 3 = 48$. The split strategy can also be utilised when working with addends by looking at their nearness to tens or doubles. For instance, if one knows that 12 is a double of 6, then they could easily work out $6 + 7$ mentally as $6 + 6 + 1$.

The scenario shown in Figure 4-57 demonstrated how learners quickly pick formal strategies without acquiring the underlying concept when formal procedures are introduced earlier (Wright et al., 2014). The learner had already represented 28 and 11 with place-value boxes, but when he was asked to find the sum, he started working it out on the chalkboard using the place-value addition algorithm. It is for this reason that Wright et al. (2014) recommend that informal strategies should be introduced to learners as soon as possible because formal strategies tend to interfere with number sense development. For instance, the example in Figure 4-57 ($28 + 11$) was also ideal for strengthening the Standard 2 learners' understanding of the split strategy for addition by asking them to mentally work it out and thereafter discuss one of the strategies as $28 + 10 + 1$.

5.5.2 Mediating talk and gesture for building mathematical connections

The interconnection of concepts is what characterises mathematics as a scientific discipline (Kozulin, 2003). Hence, the work of a mathematics teacher is centred around unpacking interconnected mathematical ideas (Ball & Bass, 2002).

One way to enable learners to discern the connections within and across examples is through the teachers' application of variation theory in the classroom (Kullberg et al., 2017; Mhlolo, 2013). However, before the learners can notice the connections, the teacher should have the capacity to notice them first and hence teach in a manner that makes the connections visible (Schifter, 2011). For young learners, Venkat and Askew (2018) emphasized that they often have limited capacity to notice mathematical connections, hence they require appropriate use of teachers' talk and gesture that makes the connections explicit.

Connections within and across example spaces

One application of variation theory involves showing the similarity or contrast within and between example spaces by focusing on their variant and invariant aspects (Kullberg et al., 2017; Mhlolo, 2013). These connections can span across topics as well as with learners' everyday lives (Anthony & Walshaw, 2009). During the first two Standard 2 lessons, the teacher enabled her learners to notice that the examples had a common sum by presenting them in pairs (see Figure 4-53) and thereafter commented about the similarity (Excerpt 4-21). After the learners had completed their group work, the connections were made visible by lining up the papers carrying the examples in front (see Figure 4-58). This was also followed by her talk about the similarity of all the answers (see Excerpt 4-22).

Whereas the Standard 2 teacher connected the examples during Lesson 1 using the ways discussed above, there were still more opportunities for making mathematical connections. For instance, the teacher used $2 + 8$ and $5 + 5$ in Episode 3 of Lesson 1, while Episode 4 had $8 + 2$,

3 + 7, and 5 + 5. The overlap of 2 + 8 and 8 + 2 from the two episodes could as well be used simultaneously in one episode to elicit the commutative property of addition. As discussed by Wright and Ellemor-Collins (2018), one way of making mathematical relationships visible is by making ordered lists. Listing 1 + 9 from Episode 2 next to 2 + 8 in Episode 3 would possibly be the first step in showing the learners what happens when one addend is increased while the other addend is increased by 1, and later extend to any other number. In the first lesson, the examples on number bonds of 10 were solved in the following order: 1 + 9, 6 + 4, 2 + 8, 5 + 5, 8 + 2, 3 + 7, 5 + 5. The same examples could have been ordered like 1 + 9, 2 + 8, 8 + 2, 3 + 7, 6 + 4, 5 + 5 to make the number relationships more visible. Furthermore, including 9+1, 7+3 and 4+6 would have enhanced visibility of the commutativity property.

In Standard 1, there were also many opportunities for strengthening connections across examples. Schifter (2011) reported that learners at Grade 1 are already able to notice the commutative property of addition. The teacher may enhance this capability of learners at this level by deliberately giving them examples in a sequence that would easily make them notice that addends can be switched around without changing the sum, such as working with 2 + 1 in parallel with 1 + 2. For the Standard 1 class, another possible connection could be linking the process of addition with the counting done during the preceding weeks. During Lesson 1, for instance, the class sang the number song four times, that involved counting from 1 to 10. This number song appeared in the teachers' guide as a teaching resource, hence could be connected to the teaching of addition (Malawi Institute of Education, 2012b).

Some of the examples given by the Standard 1 teacher were repeated. For example, 2 + 1 was repeated in all the six lessons. During Episode 4 of Lesson 4, 2 + 1 was part of the individual work, yet the same 2 + 1 was also part of group work during Episode 3, and the answer had just been verified by the whole class. The repetitions could possibly be utilised to strengthen

number relationships. The relationship among the numbers 0 to 5 would be made possible by the systematic ordering of the numbers and their commutative pairs like as follows:

$0 + 1, 1 + 0, 1 + 1, 1 + 2, 2 + 1, 1 + 3, 3 + 1, \dots$

$0 + 2, 2 + 0, 2 + 1, 1 + 2, \dots$

$0 + 3, 3 + 0, 3 + 1, \dots$

$0 + 4, \dots$

$0 + 5, \dots$

In addition to making number relationships visible, the systematic ordering also generates more examples within the range 0 to 5.

Connecting various means of mediation

The teachers demonstrated remarkable use of multiple physical and visual representations within the same tasks, thereby decompressing and compressing mathematical ideas in many ways (Ball et al., 2008). The Standard 1 teacher, for instance, trained the learners to pay attention to the movement of a pointer to enable them to connect her gestures to the inscriptions on board. All the teachers involved the learners in reading and writing the chalkboard inscriptions. This made it possible for them to make connections between the physical and written representations of the same mathematical concepts.

When using place-value boxes, the Standard 2 teacher strengthened the connections between written and physical representations of the same addend by putting the place-value box below the written number, as shown in Figure 4-54. After adding two numbers using place-value boxes, the class was asked to find the sum by interpreting the number represented by the total number of bundles and sticks. This was done by counting the tens and ones in the box carrying the sum. Asking learners to tell the number represented by the bundles and single sticks enabled them to switch back and forth between the physical and verbal representations of the same concept of number. After verbalising the sum, learners were invited to write the sum on the

chalkboard. This made learners connect the verbal representation of the number with its written inscription. The back and forth switching between representations done by Standard 2 teacher is what Ball et al. (2008) calls decompressing or unpacking mathematical ideas into a simpler form, and packing the concepts again into their original compressed mathematical form.

The teachers extensively used gesture corresponding to the utterances made during the lessons. For example, the Standard 1 teacher used special gestures for denoting “zero” and “plus” or “add”, and taught learners specific hand movements for writing the numbers 0-5, as well as the + and = signs (see section 4.3.4). In some instances, she used hand clapping and asked learners to clap their hands once and to clap zero times, representing the numbers 1 and 0.

Use of language

Language is one of the key sociocultural tools that provides a medium for conveying mathematical concepts to learners (Essien, 2018; Kozulin, 2003). Since language characterises the teacher’s talk in the classroom, proper use of language can have an effect on learners’ understanding of concepts, thereby affecting achievement (Davis et al., 2015; Graven, 2016).

Language issues affecting the teaching of mathematics

There have been so many language-related issues that affect the teaching of mathematics to learners in their early years of primary school in Malawi and other countries in Sub-Saharan Africa (Chitera, 2012; Essien, 2018; Kaphesi, 2003; Kazima, 2008). Despite the prevalence of language issues affecting learners, Anthony and Walshaw (2009) indicated that teachers of mathematics may sometimes not be aware of what their learners are going through. The awareness of the teacher can have positive outcomes. For instance, in their meta-analysis of international research on small group work, Walshaw and Anthony (2008) reported a study on practices in a Grade 2 mathematics classroom that indicated much conceptual gain when the

teacher took note of learner's interpretations and let the whole class use the same quality of explanations given by their peers.

As for this study, it was observed that learners had the freedom to use their expressions during group work, but they were often reminded to use the standard terminology used by the teacher when presenting their work to the whole class. For instance, when performing place-value addition involving regrouping, some Standard 4 learners stated that they carried over a digit to the next place-value because "you cannot have two digits under one place value, hence the other digit has to be moved to the next place value", while the standard explanation given by the teacher had the form: "A digit representing a ten cannot be placed under ones, hence it has to be moved to other tens". The learners' explanation always worked without worrying whether the digit is a ten, a hundred, or a thousand. In another case, a learner referred to "hundreds" using an equivalent Chichewa slang "mahanzi" that was popular among the youth. This seemed to have excited the class, but the teacher was quick to ask the learner to use the correct Chichewa term "mahandiredi" that she claimed "the class understands", which sounded ironical. The irony came in that the acceptable Chichewa terms for the place-values that the class was said to understand did not have a visual connection with the English place-value notations (Th, H, T, and O) that were used, as discussed in the following subsection. The teacher, however, ensured that the learners should not shift from the formal terms used in mathematics.

Translation of mathematical terms

In the early years of primary school in Malawi, the language of learning and teaching is Chichewa (Kazima, 2008). Ordinarily, the translation of terms from one language into another language faces challenges that often result from contextual variations across cultures (Ng et al., 2012). During the study, there were notable challenges related to the teachers' use of common terms, starting from the Chichewa word for "number". For instance, the Standard 2 teacher

referred to written addition statements such as “ $12 + 5 =$ ” or “ $3 + 9 =$ ” using a singular reference, *nambala imeneyi* in Chichewa, which means “this number”. The teachers’ use of the Chichewa word “nambala”—which is used as a borrowed word from English, like other words such as *tebulo* for table, *kapu* for cup (Kazima, 2008)—had several inherent language-related issues. One possible complexity associated with the word “nambala” is the variation among native speakers on the singular and plural renderings of the word. The Chichewa word “nambala” relies on its use in a sentence to reveal whether it is in singular or plural form, hence “nambala” could also mean “numbers”. The confusion comes in that some native speakers tend to put a prefix *ma-* in the plural form—thus, changing it to *manambala* as the plural rendering for “numbers”.

Going back to the Standard 2 teacher’s reference to addition statements like “ $12 + 5 =$ ” or “ $3 + 9 =$ ” as “this number”, there is a possibility that the teacher’s intended meaning was different. Since the written addition statements that were being referred to had two numbers, the possible assumption is that the teacher ran short of the equivalent Chichewa terms for “expression” or “statement”. Taking Excerpt 4-19, for instance, there is a possibility that usage of the term “number” by the teacher in Utterances 35 and 41 (when referring to “ $3 + 9 =$ ”) evoked a conceptual connection with a single value to some learners. As such, Learner 7 in Utterance 38 might have been tempted to give out the answer 12—a “number” obtained from the given addition statement. Contrary to the learner’s thinking, the teacher’s use of the term appeared to refer to the entire mathematical statement. Towards the end of the lesson, the teacher seems to have avoided uncertainty in the use of “nambala” by just referring to the addition statements as “first one” and “second one”. Another possibility was for the teacher to use the Chichewa word *samu* transliterated from “sum” to mean the statement “ $3 + 9 =$ ”. This could have clarified the confusion considering that “samu” is the popular rendering of “a mathematical problem” in Chichewa, whether the problem is on addition or otherwise. It is for this reason that the

whole subject of mathematics is called *masamu* [sums]. The teacher used the word “sum” during the opening of Lesson 2 when asking the learners what “sums” were learnt the previous day. The Standard 1 teacher used the Chichewa phrase *chiganizo chophatikiza* as a direct translation of “addition sentence” used in the teachers’ guide. However, the meaning of “chiganizo chophatikiza” is not obvious and the teachers are supposed to explain it to learners.

The Standard 3 teacher, on the other hand, used the word “nambala” in Excerpt 4-30 with another connotation that also resulted into some misunderstanding for the learners. In the excerpt, the teacher had just started to work out $541 + 27$ with the class, and was starting to build the first abacus for 541. The teacher wanted to ask the class to mention the digit under the place-value heading for ones, for the first addend, 541. But since the Chichewa has no word for digit, a possible Chichewa rendering for the word “digit” could be *dijiti*. Instead, the teacher used “nambala” for both number and digit. The teacher asked: “... Along the ones, how many numbers are there?” This made the learners think of 1 and 7 and answered “Two!” To solve the communication problem, the question was changed to: “...the first number....along the ones, how many things are there?” to mean “along the ones, what digit do we have?” By asking, “how many things are there?”, the learners quickly understood the teacher’s question because at that moment they were looking for the number of counters to be placed along the ones spike of their abaci. As such, the learners understood the teacher’s question as saying: “How many things should be placed in the abacus along the ones?”

The teachers were compelled to use the terms provided in the teachers’ guides or learners’ textbooks even if they would not make sense to the learners at that particular moment. For example, the teachers taught the place-value heading “O” as *mawani* [ones] in Chichewa even though there is no “O” in the Chichewa word, as it is in “ones”. The teachers had to teach it just like any other symbol hoping for the learners to understand the conceptual meaning behind the headings when they switch to English later in Standard 5. It is sometimes argued that the

use of transliterated terms, though not making sense during the early years, they make the switch to English easier later (Kazima, 2008). Teachers faced these challenges because the teachers' guides are always provided in English while the teaching is done in Chichewa. Also, primary school teachers in Malawi do not critique the teacher's guide or learners textbooks (Kazima et al., 2016), as discussed in section 5.2.1. Even though learners' textbooks are in Chichewa, they mostly just provide a list of problems for learners to solve. What adds to the challenge is that teacher training colleges for primary use English as the medium of instruction, hence not prepare the teachers adequately to teach in Chichewa (Chitera, 2012).

5.5.3 Mediating talk and gesture for advancing learning connections

This section focuses on how the four teachers worked with learners' ideas. Usually, the errors made by learners during class discussions may present some opportunities to teach—or “teachable moments” (Muir, 2008, p. 362)—that would not have been known to the teacher if the errors had not been made. This, in turn, elicits the teacher's “responsive moves” (Venkat & Askew, 2018, p. 80) to re-direct the lesson to aspects that may enhance the learners' conceptual understanding or clear some misconceptions, even if this was not planned by the teacher. Thus, learners' responses signify the right moment where they may be more receptive to the teacher's explanation.

5.5.4 Verifying of learners' offers

After a learner had given an offer, the teachers generally asked the class if it was correct. If it was not correct, they mostly asked a different learner to give another offer. In some cases, the teachers probed the offers further to strengthen learning connections.

In Standard 1—while discussing solution to the problem $3+2$ written on a piece of paper and was read by one learner as “three plus two answer four”—the class was divided on whether the inscription that the learner had written and called “four” (see Figure 4-38) was written

correctly or not. This disagreement sparked a teachable moment. The teacher corrected the error in Figure 4-38 by asking another learner to come and write another 4 on the chalkboard (see Figure 4-39). The teacher considered it necessary to start with remediating the writing of 4 before checking whether the sum given by the learners for 3 and 2 was indeed 4. This was deemed the right moment to focus on 4 probably because the learners would be more receptive. The teacher proceeded with the use of similarity and contrast to show both a correctly written 4 as well as a wrongly written 4. As the criteria for checking the correctness of writing 4, the teacher used the verbalised hand movement (“Dot! Down! Turn-right! Cut-in-the-middle!”) that the learners were taught when they were being introduced to the writing of 4 in the air. After discussing the correct and incorrect ways of writing 4, the teacher and the class proceeded to work out $3 + 2$, to check if its answer was really 4. A study by Aploon-Zokufa (2013) showed that learner performance was higher when the criteria used for evaluating learners’ responses was understood by the learners.

During Lesson 3 of Standard 2, learners took up to three attempts to write 39 as the answer for $28 + 11$ on the chalkboard. The first two learners wrote 29 and 59 respectively before the third learner wrote 39. After each attempt, the teacher asked the class if it was correct, followed by asking what number it was, and ended by asking what they are looking for. The teacher always gave the class the power to decide whether the solution given was correct or not. Instead of just requiring yes or no from the class, the teacher asked the class to say why the given answer was not correct, and what can be done for it to be correct. The number of incorrect attempts made by the learners signifies why the teacher always asked the learners to read the numbers given on the chalkboard and to write solutions found. In so doing, the teacher continued offering learners more opportunities for making connections between the current lesson and previous lessons on counting and writing numbers between 20 and 50. This was further evidenced by the frequent repetitions that were more during Lesson 3 of Standard 2 than lessons 1 and 2.

5.5.5 Positive reinforcement

The teacher motivated the learners with positive reinforcement techniques, such as letting learners decide what hand-clapping style they would prefer (Excerpt 4-29). In one discussion involving all the four teachers, they agreed that the CRECCOM hand-clapping style was the most valued of all claps and would normally be given to a learner in special instances. Other reinforcements included a chief's clap, and Mandela wave. The Standard 4 teacher included humour by asking the learners to make sounds such as a train or a maize-mill when praising a learner who had given a correct offer.

5.5.6 Paradoxical teaching

The Standard 4 teacher exemplified how to handle one of the dilemmas faced by a mathematics teacher highlighted by Ball (1993), that is, inducting the learners in advancing ideas based on reasoning and mathematical arguments rather than resting on the authority of the teacher. Ultimately, the learners should get used that the role of their teacher is not to establish the validity of solutions. The Standard 4 teacher deliberately made mistakes for her learners to identify the mistake and correct her. Oftentimes, the teacher caught her learners unawares by leading them to a logically wrong conclusion that looked appealing to them. In some instances, it took only a few seconds for the learners to realise that the teacher was playing a trick leading them to an incorrect solution. This meant that whenever the teacher had asked "alright?" as she solved the problems, the learners had to be very attentive because if not careful, they might say "yes" where they should say "no". For instance, after one had worked with the regrouping algorithm with a series of 4-digit numbers, by the time they reach the thousands they may be tired and may possibly be happier to work with smaller digits during the last steps of the process. The teacher took note of this, hence in Excerpt 4-48, after finding a small sum of 5 from the thousands she said: "We have found that it is 5. Here, do we have to bother with keeping anything?" and then proceeded to write 5 under thousands on the chalkboard. In other

words, the teacher resonated with common thinking and was actually speaking the mind of the learners, who might think: “Why trouble oneself consulting the ‘carry-over inscriptions’ from the hundreds while the obtained sum of thousands is less than 10?” The learners later realised that they were being lured into a trap and rejected the teacher’s offer. The teacher’s point was that regardless of the fact that they had reached the highest place-value and had found a small sum, they needed to check if they had a carry-over digit from the preceding place-value and add it before writing down.

The teacher’s approach seemed to have enhanced the learners productive disposition (Kilpatrick, Swafford, & Findel, 2001), considering themselves as doers of mathematics. For instance, during Lesson 2, the Standard 4 teacher was occupied with marking the last few notebooks when one of the learners was writing one of the solutions on the chalkboard. The class went ahead and approved the answer, and just informed the teacher about the outcome.

5.6 Experiences of using the MPM analytical framework during the study

The methodology chapter presented some dilemmas that shaped the use of the MPM framework in this study. Even though the challenges associated with scoring were addressed in the methodology, some of the issues were also carried over when presenting the findings and discussion.

5.6.1 Dilemmas from seeming overlaps between means of mediation

As discussed in the methodology chapter, analysis of the lessons using the MPM framework was affected by the blurred distinctions between some of the forms of mediation. This would not be surprising considering the sociocultural background of the four strands of mediation (tasks and examples, artefacts, inscriptions, and talk and gesture) identified by Venkat and Askew (2018).

Identification of artefacts and inscriptions

When presenting findings, it was challenging to classify some observed means of mediation as either artefacts or inscriptions. As stated by Venkat and Askew (2018), the key features for distinguishing artefacts from inscriptions are whether they are teacher-generated and whether they are brought into the classroom (pre-made), and their existence afterwards (permanence). Based on this, most of the prewritten papers observed during this study were classified as artefacts. However, looking at how these prewritten papers were used, they oftentimes befitted the teachers' use of inscriptions. For instance, as shown in Figure 4-26, the Standard 1 teacher worked with examples written on chart-sized papers in the same manner as she would use the chalkboard. Even though the papers were prewritten and had the property of permanence, the mediating inscriptions for working out the solutions on the papers were always generated in the course of the lesson.

Teachers' use of inscriptions

Another challenge with the MPM framework was associated with the use of inscriptions. The MPM framework mainly focuses on the nature of inscriptions by examining the extent to which they are structured or not structured (Venkat & Askew, 2018). Although the framework explicitly presents the usage of inscriptions, it does not explicitly provide the means for describing the use of inscriptions. Since inscriptions are mainly characterised by their temporary usage during the lessons, just like talk and gesture, the MPM framework might have emphasized on their mediatory role rather than how they are mediated by the teacher's talk and gesture. There were instances during the study where the inscriptions were mediated by artefacts, as was the case with place-value boxes and spike abaci. During the study, however, it was possible to isolate how the teachers used inscriptions as discussed in section 5.4.

Use of artefacts, and teachers' talk and gesture

As shown in sections 5.3.3 and 5.3.6, the teachers' use of artefacts could not be discussed in isolation from the other strands of the MPM framework. The use of artefacts was often closely linked to the teachers' mediating talk and gesture for providing methods for generating solutions to problems as well as mediating talk and gesture for building mathematical connections. This made it challenging to discuss the teachers' use of artefacts adequately to avoid the seeming overlaps with teachers' talk and gesture.

5.6.2 Lessons learnt from the application of some analytical assumptions

Since during the course of the research project, there were not many studies exemplifying usage of the MPM framework in the classroom, usage of the framework was understood in practice. For instance, their analysis of exemplar lessons to illustrate the usage of the framework, both Venkat and Askew (2018) and Askew (2019) mentioned analysis of learners' errors as a key factor determining whether to analyse an episode or not. These episode-by-episode checks would ultimately deem a lesson worthy of being interrogated further using the framework or not (Askew, 2019). The assumption by Venkat and Askew (2018) was that errors from learners' offers signify whether the lesson had new concepts being taught for the first time or it was basically a rehearsal of previous learning.

During the study, there were minimal errors from learners during the first two lessons of Standard 2 that focused on number bonds of 10 and 12. In addition to having minimal errors from learners' offers, the two lessons flowed exactly in the same manner, as illustrated in Table 4-6 to Table 4-8 in section 4.4.2. Based on the assumptions from Venkat and Askew (2018) these lessons would not have been analysed. Despite this similarity, all the five episodes in each of the two lessons were analysed, focusing on new opportunities of teaching that emerged during the teacher's interaction with the learners in the classroom, instead of the traditional

focus on opportunities of learning. In the end, the findings from Lesson 2, though not different from Lesson 1, opened more insights into the teacher's use of language and its possible influence on learners' comprehension of the teacher's talk.

In support of the arguments by Venkat and Askew (2018), however, Lesson 2 signified how a repeated lesson would greatly limit new learning opportunities to learners. As such, Lesson 2 gave additional meaning to Askew's (Askew, 2019) notion of a repeated lesson. Lesson 3, on the other hand, was centred around the representation (modelling) of the concept of addition using place-value boxes. This way of modelling the process of addition was being taught for the first time to the learners. The findings from Lesson 3, gave some insights into the possible relationship between the novelty of a concept being taught in the class and the richness of the responsive moves (Venkat & Askew, 2018) made by the teacher. During Lesson 3, there were many movements made between representations of the same concept. Learners also worked back-and-forth between representations: composing or modelling numbers with place-value boxes and "reading" numbers physically represented by the place-value boxes. Of all the three Standard 2 lessons observed, Lesson 3 had the richest usage of gesture by the teacher. The lesson also had considerable use of repetition by the teacher compared to Lessons 1 and 2. Overall, Lesson 3 contributed much more to the study, making it worthy of the analysis that was done (Askew, 2019).

5.7 Chapter summary

This chapter discussed the usage of mediational means by the four teachers. As regards their selection of tasks and examples, the teachers mainly followed the guidelines specified in the teachers' guide. As such, the limitations in their selection of tasks and examples were sometimes influenced by the constraints in the source documents that they used. There were incidents where teachers provided more learning opportunities by going a step further from the

instructions provided in the teachers' guide. The teachers also appeared to believe in the teaching potential of artefacts despite requiring their efforts to make them. The teachers used the artefacts for ensuring learner engagement throughout the lessons. The popularity of the framed counter signified the potential of modifying it slightly to make it a mathematical tool for promoting competencies such as subitizing among learners. The teachers also used chalkboard inscriptions for presenting tasks and for showing methods followed to arrive at the required answer. Chalkboard inscriptions were also used for promoting learner engagement. The teachers provided several opportunities for making mathematical connections through their use of talk and gesture. Since the limitations in the teachers' use of talk and gesture were common among all the teachers, there is a possibility that the observed usage patterns were inherited from their training or from the curriculum materials that they used. Finally, the chapter discussed the experiences of using the MPM analytical framework during the study and highlighted the dilemmas from seeming overlaps between means of mediation, and the lessons learnt from the application of some analytical assumptions.

CHAPTER 6

CONCLUSIONS AND IMPLICATIONS

6.1 Introduction

This chapter presents the major insights from the study and their implications to the theory and practice of teaching mathematics to learners during the early years of primary school in Malawi. Mathematics teaching for young learners was understood in terms of the mediatory role of the teacher that is accomplished through tasks and examples, artefacts, inscriptions, as well as talk and gesture (Venkat & Askew, 2018). As such, the first area of focus for this study—represented by the first research question—was how the teachers selected the tasks and examples. The second question focused on how the teachers used artefacts, inscriptions, and explanations to convey the mathematical concepts and processes associated with the selected tasks and examples. The last question examined the rationales behind the teachers' choices of the mediational means used during the lessons.

The chapter starts with a presentation of the major findings in section 6.2 followed by implications of the findings, contribution to knowledge, limitations of the study, suggestions for further research, and ends with a reflection on my personal growth.

6.2 Summary of the major findings on teachers' usage of mediational means in the early years of primary school classrooms

Despite the differences among the individual teachers' overall experience in teaching mathematics in the early years of primary school classes, their usage patterns of mediational

means shared several similarities. This section summarises the findings for each of the three research questions, starting with the first research question on how teachers select tasks and examples.

6.2.1 Teachers' selection of mediating tasks and examples

The teachers presented the lessons using the same pattern in their sequencing of tasks from the introduction to the conclusion. They generally started with an example discussed by the whole class followed by some examples done through group work, and ended with examples done as individual work. The teachers selected tasks and examples based on the instructions from the teachers' guide. The teachers worked with the examples presented in the learners' textbook except for one teacher who formulated her own examples. One factor guiding the teachers' selection of examples was their difficulty, ensuring that both simple and difficult examples are included. The teachers also ensured that the sum of the addends does not exceed the number range specified in the syllabus.

The ordering of examples within and across tasks was random. Some of the examples selected by the teachers had the potential for showing connections if they had been sequenced in such a way that systematic patterns would be made visible to the learners, with accompanying teacher talk emphasising on such relationships. The random ordering of examples was also noted in the teacher's guide. As such, the teachers had no basis to apply some aspects of variation theory in their choice of examples.

6.2.2 Teachers' use of mediating artefacts, inscriptions, talk and gesture

This subsection addresses the second research question on how teachers use artefacts, inscriptions, and explanations to represent mathematical concepts and processes.

Teachers' use of mediating artefacts

The teachers used both unstructured and structured artefacts in their lessons. Unstructured artefacts included counters while structured artefacts included place value boxes and spike abaci. The artefacts could also be classified as single use or multiple use artefacts. The single-use artefacts, such as leaves, sticks, and stones, were mainly brought to class by the teacher while the multiple-use artefacts, such as framed counters, were developed by the learners at home and brought to class every day. Single-use artefacts were mainly used in Standard 1 and were used for counting numbers and for (modelling) counting all strategy in addition of numbers.

The most popular artefact used across all the classes was the home-made framed counter. The teachers made the use of framed counters mandatory among the learners. This made the use of artefacts appear to be at the core of the teachers' mediation strategies in almost all the lessons—but more extensively used in Standard 1 and least used in Standard 4. The framed counters were mostly used for (modelling) addition of numbers using counting all strategy. The mapping of numbers with physical manipulation of counters when working out solutions made the learners experience mathematics as a physical activity rather than a solely cognitive undertaking (Edwards et al., 2014). The repeated use of physical manipulatives helped to maintain learners' attention during the lessons. Prewritten papers that were left on the wall after a lesson gave an opportunity to learners to see the work done during the previous lessons.

Teachers used multiple representations of the same concept. In some cases, this was done by flexible movement from the physical representation of a mathematical concept or process to its abstract representation and vice-versa. This was good because a single model may not capture all aspects of a mathematical idea—as Ball (1993) observed that teachers in the early years classes are faced with the dilemma of choosing a representation for a concept. The teachers of Standards 2 and 3 worked with artefacts that carried the structural properties of numbers. These

structured artefacts were place-value boxes in Standard 2 and the spike abaci in Standard 3. The place-value boxes used in Standard 2 physically represented two-digit numbers using bundles of ten sticks for tens and single sticks for ones. Even though the place-value box worked well with two-digit numbers, its use for larger numbers in Standard 3 might have been deemed demanding. As such, the curriculum materials shifted to the use of spike abaci for representing structural properties of three-digit numbers in Standard 3.

Teachers' use of mediating inscriptions

The major use of inscriptions by all the teachers was for the presentation of tasks and examples on the chalkboard. The teachers mostly presented incomplete addition statements and asked learners to complete the inscriptions leading to the solution. Inscriptions were also used for showing the method for arriving at the solution. In Standards 3 and 4, inscriptions for showing the method were presented using arrows and auxiliary calculations written below the problem. Some of the teachers also used inscriptions for the reification of mathematical processes (Sfard, 2008). This was more noticeable in the way the Standard 1 teacher presented tasks. The reification was achieved both within and across examples during the lessons. The Standard 1 teacher started by presenting tasks using concrete objects during the first lesson and progressively shifted to the sole use of structured addition statements during the last three lessons. The Standard 2 teacher also reified the process of two-digit addition by using multiple inscriptions in varying levels of structure during her last lesson. During the last lesson, the Standard 2 teacher started by presenting each addend as a plain number, followed by rewriting the addends under place-value headings, ending with structured mathematical statements in column addition format.

Teachers' mediating talk and gesture

The teachers provided opportunities for learners to solve problems on the chalkboard—thus created a learning community of mathematical discourse in the classroom (Ball, 1993)—which also created many opportunities for mediating talk and gesture. The Standard 4 teacher exemplified how learners might be inducted into the classroom discourse by letting them be the ones guiding the teacher on the next step to take, and corrected her whenever she pretended to make common mistakes. In so doing, the learners were given more opportunities for expressing their arguments, thereby develop their mathematical thinking and enhance a positive mathematical mindset (Boaler, 2016). By letting learners present their arguments, the teachers respected them as mathematical thinkers (Ball, 1993).

The teachers responded to learners' errors, thereby building learning connections. Even though this required more time, the time spent on “teachable moments” (Muir, 2008, p. 362) elicited by learners errors provided more and richer learning opportunities than what the teacher might have planned to teach. During such times, the learners' errors revealed their misconceptions and signified the most probable time when the learners were more receptive to correct the misconception and make more mathematical connections (Muir, 2008).

The teachers also structured their talk in such a way that they would sustain young learners' attention throughout the lessons. There were no long explanations of the mathematical concepts and processes, instead the teachers' talk was comprised of short statements that were mostly phrased as questions. As such, at every moment, learners were expected to follow what was being said or written on the chalkboard to know how best to respond to the teachers' series of questions. In all the lessons, the teachers attempted to engage learners when generating solutions to problems through unit counting. However, the repetitive unit counting processes that were used when doing addition seemed simple for the learners at the beginning when

adding small numbers, but became more cumbersome and time consuming as they re-used the unit counting processes many more times for addition of larger numbers.

In some cases, the teachers faced challenges with the use of language when referring to certain mathematical terms. The major challenge was that the teachers' guide and syllabus were in English even though the language of learning and teaching was Chichewa. As such, the teachers sometimes had to figure out on the spot what term they would use to refer to a mathematical term or concept. For instance, the teachers worked with the place-value notations Th, H, T, and O in Chichewa even though they are based on English spellings of the place-values. The teachers presented Th as *masauzande* [thousands], H as *mahandiredi* [hundreds], T as *mateni* [tens] and O as *mawani* [ones] and learners memorised them even though they could not yet see the connection between the notation and the Chichewa name, especially for Th (*masauzande*) and O (*mawani*).

The next sub-section focuses on the third research question on the rationale behind the teachers' choice of examples, artefacts, inscriptions, and explanations used during lessons.

6.2.3 The rationale for the teachers' choices of mediational means

The teachers' selection and sequencing of examples was highly influenced by the instructions given in the teachers' guide. As such, the quality of the teachers' utilisation of the selected tasks and examples to show mathematical connections could possibly be attributed to the quality of the instructions given in the teachers' guide. Some of the teachers made slight changes by adapting the instructions in the teachers' guide to show connections within and across examples. For instance, the Standard 1 teachers guide asked the teacher to demonstrate how to write $2 + 1 = 3$, but the teacher decided to challenge the learners to attempt writing it by themselves. As the learners were taking turns attempting to come up with $2 + 1 = 3$ on the chalkboard, the teacher noticed how her talk influenced what the learners were writing on the

chalkboard and adjusted it accordingly to enable the learners reach the desired goal. In some cases, the teachers' guide asked the teachers to work with various mediational means in separate examples, but the teachers decided to work with them simultaneously using the same examples. The simultaneous use of various means of mediation for the same examples presented more opportunities of learning and saved the time that would have been spent on doing separate examples.

Class management also seemed to influence the teachers' choice and use of mediational means. The teachers always insisted that every learner should use their counters or fingers as a way of promoting learner engagement. When working on the chalkboard, the teachers also engaged learners at every stage of the process leading to the answer. By engaging all learners, the teachers managed their large classes well.

6.3 Implications of the findings

The findings point to some implications for the early years' mathematics curriculum, teaching and monitoring, as well as teacher education.

6.3.1 Implications for mathematics curriculum materials

Since the study found that the teachers selected tasks and examples from textbooks and they follow the teachers' guide when teaching, it can be assumed that the opportunities of learning afforded by the selected mediational means could mainly be linked to the source documents used by the teachers. The teachers worked with examples that had a potential for showing mathematical connections if they were presented in a way that would make the number properties and relationships visible. Teachers might teach for connections if the number relationships were also made explicitly visible in the curriculum materials that they use. For instance, it is not surprising that the teachers did not present word problems that develop

alternative strategies for addition considering that all the problems given in the textbook were formulated using the ‘combine and count all’ approach.

The curriculum was structured in a way that learners were adding numbers that they had just learnt to write during the preceding weeks, making the teachers handle a dual task of mediating the process of addition and at the same time familiarising the learners with the new numbers. As such, some learners who did the calculation properly could not manage to write the answer correctly. The teachers’ work would have been made easier if the curriculum was structured in such a way that learners are taught as many numbers as possible at the outset followed by teaching them as many strategies as possible for carrying out operations on the numbers. For instance, instead of limiting the range of numbers from 0 to 9 in Standard 1, the learners can be exposed to counting numbers up to 100 but limit the writing to a lower range, such as 0 to 20.

6.3.2 Implications for teaching

The teachers’ use of multiple means of mediation for representing one concept exemplified teaching for connections. The study, however, showed that the connections are weakened when learners are quickly introduced to formal algorithms of addition before being made thoroughly familiar with the conceptual representations of the same. When teachers bring in an alternative representation of the formal algorithm, such as the use of an abacus, it might be too late for the learners to make sense of the conceptual reasons behind the formal algorithms. As such, some learners make significant mistakes without realising the errors by themselves. This implies that mathematics teaching has to focus on enhancing number sense development through informal strategies for addition before exposing the learners to the formal algorithms.

Regardless of the means of mediation used, the teachers used them for learner engagement. Whether working with artefacts or inscriptions, the teachers asked learners to do more of the

work thereby keeping them active, and at the same time covering more examples. This implies that large classes can discreetly be utilised for synergy, thus covering more content, as did the teachers in the study.

6.3.3 Implications for teacher education and development

The study found that all the lessons by the four teachers followed the same structure, regardless of the years of experience of teaching and of teaching mathematics. In many instances, counters were used when not necessary, making learners stick to the use of physical artefacts instead of letting them work flexibly between representations. It was found that the teachers' extensive use of counters seemed to be guided by a general belief in the use of concrete manipulatives as the major means through which young children can understand mathematical concepts. This suggests that the teachers retain the mediation skills learnt during teacher education. Hence, strengthening the teacher education could possibly have a noticeable effect on classroom teaching practices.

This points to the need for teacher education that focuses on developing metarepresentational competence in learners—which is the ability of letting learners choose alternative representations based on efficiency or ease of use for a particular task (diSessa, 2004). The first step in enabling the teachers to show connections between representations during lessons is for themselves to notice the conceptual relationships during lesson planning. It has been said, however, that teachers cannot start noticing relationships across mediational means on their own. As observed in a study by Goldsmith and Seago (2011), teachers can only start noticing opportunities for eliciting learners thinking through teacher education or professional development (PD) programmes aimed at such. For instance, familiarizing teachers with the basic principles of variation theory (Kullberg et al., 2017) could enable teachers to start noticing the horizontal and vertical connections across examples that appeared to be lacking during most of the lessons. Such training would also include school inspectors and advisors, so

that they would not always expect the use of concrete manipulatives when inspecting every early-years' mathematics lesson.

6.4 Contribution to knowledge

In general, the study contributes to knowledge on teachers' mediation in mathematics classrooms in early years by providing research findings from Malawi. The originality of the research and findings in Malawi context contributes to knowledge in the field.

Specifically, the study's main contribution is on the use of the MPM framework to study the teaching of mathematics to learners in the early years of primary school. While the framework made it possible to examine the teachers' mediating actions to the finest detail, it needed some modifications to make it function optimally. This study therefore contributed to the refinement of the framework.

6.4.1 Usage of the MPM framework by a single researcher

This study exemplified the usage of the MPM framework by a single researcher. By the time the study was being conducted, the published applications of the MPM framework exemplified its use by a team of researchers in a PD setup (Askew, 2019; Askew et al., 2019; Venkat & Askew, 2018). The major usage aspect of the framework favouring teams is the numerical levelling that is aimed at measuring and comparing the quality of mediation across different lessons by the same teacher or by different teachers. This study demonstrated that the descriptive elements of the framework are still usable by a single researcher interested in exploring and examining the usage of mediational means by a set of teachers outside a PD setup.

Resolving scoring dilemmas

The scoring dilemmas associated with a single researcher's use of the MPM framework were reported in section 3.5.6. For instance, when two artefacts are used concurrently in an episode,

where one is used in structured ways and the other is used in unstructured ways, it was hard to determine the overall final score. This was the case when unstructured use of counters was coupled with structured use of prewritten papers in the same episode during the study. This dilemma was also experienced where the usage of particular mediational means appeared to be partially structured, that is, mid-way between structured and unstructured use. It could not be easily established from the framework whether a score such as 2.5 was possible, though the calculations done by Askew et al. (2019) signified that the scoring levels could be used beyond the ordinal scale of measurement.

These dilemmas when using the MPM analytical framework were resolved by focusing on the usage indicators provided in the framework rather than the numerical scores. The analysis focused on describing the mediation rather than assigning numerical scores to the observations. Therefore, this study has contributed by extending the framework to include condensed descriptions of mediation to facilitate coding without the use of numerical scores (see Table 6-1).

Mediational overlaps

There were some challenges in coming up with distinct categories of mediational means. For instance, a discussion of the use of mediating artefacts seemed to overlap with the teacher's talk for generating solutions to problems. The next sub-section summarises some of the usage experiences of the MPM framework in comparison to the theoretical assumptions based on the way the framework was understood.

6.4.2 Working with theoretical assumptions governing the analytical framework

The MPM analytical framework assumes that if incorrect offers or inefficient methods are not identified in an episode, then the episode is possibly a rehearsal of previous learning, hence it is not analysed (Askew, 2019; Venkat & Askew, 2018). Since this study focused on the

teachers, the analysis done in this study examined all the episodes regardless of the feedback from learners. Thus, the findings from the study strengthened the argument by Venkat and Askew (2018) on the mediational similarity of seemingly repeated lessons and, at the same time, revealed some opportunities of teaching in the episodes that would have potentially not been discussed if the episodes were skipped based on learners' offers.

Relationships within and across example spaces

The MPM framework considers tasks and examples as the foundation upon which mathematics teaching is overlaid. The assumption is that examples and tasks are mediated through the other three strands, hence cannot be analysed separately. As such, examples are only listed in the MPM framework, and the analysis focuses on how the examples are mediated by artefacts, inscriptions, talk and gesture. Since this study was aimed at exploring how teachers worked with examples, it came out that the discussion on how the teachers worked with the connections within and across example spaces was done under the teachers' mediating talk for building mathematical connections.

Use of mediating artefacts

Structured use of either structured artefacts or unstructured artefacts is accorded the same highest rank in the MPM framework. This could imply that whether artefacts are structured or unstructured, their use by the teacher is more significant. During the study, the use of artefacts overlapped with the teacher's talk and gesture for generating solutions to problems.

Use of mediating inscriptions

Regarding mediating inscriptions, the MPM framework considers them as temporary, and the use of inscriptions was not coded in the framework. The framework only provided for the type of observed inscriptions. This might suggest that inscriptions only mediate but cannot be mediated. During the study, there were many incidents where the teacher's talk was directed

to the inscriptions, such as asking learners to read them or append to the existing inscriptions. This could not be easily coded directly using the framework as the use of inscriptions because the framework only provides for the type of inscriptions. However, since inscriptions are dynamic in nature, it can be expected that inscriptions can mediate other inscriptions. This study demonstrated how inscriptions were used to mediate other inscriptions, such as arrows that illustrated the place-value algorithm. Thus, contributed to the framework by exemplifying instances that signified the *type* and *use* of inscriptions.

Mediating talk and gesture

Some aspects of teacher talk overlapped with the use of artefacts and inscriptions as described above. The study demonstrated how the various forms of teachers' mediating talk and gesture could be segregated from the discussion of other mediating means. Thus, again, contributed to the framework.

6.4.3 Suggested modification to the MPM framework

Table 6-1 illustrates the modifications made to the framework presented in Table 2-1, showing the tasks and examples strand spanning across the other three strands, at the same time showing an overlap with talk and gesture for building mathematical connections.

Table 6-1: Condensed coding scheme for MPM framework (Adapted from Venkat and Askew (2018, p. 90))

Tasks and examples				
Artefacts	Inscriptions	Talk and gesture		
		<i>Method for generating solutions</i>	<i>Building mathematical connections</i>	<i>Advancing learning connections</i>
No artefact or incorrect (NA)	No inscription or problematic (NI)	No method or problematic (NM)	Problematic examples (PE)	No evaluation of offers (NO)
Unstructured artefact & unstructured use (U _u A)	Recording-only inscriptions (RI)	Singular method (SM)	Redundant examples (RE)	Evaluates offers (EO)
Unstructured artefact & structured use (U _s A)	Unstructured inscriptions (UI)	Localised method (LM)	Single-connection examples (SE)	Verifies offers (VO)
Structured artefact & structured use (SsA)	Structured inscriptions (SI)	Generalised method (GM)	Multi-connected examples (ME)	Justifies offers (JO)

Whereas the MPM framework only lists examples without discussing them separately, the overlap shaded in Table 6-1 was utilised to discuss the nature of the selected examples.

6.5 Study limitations

There were two limitations of the study, as discussed below.

6.5.1 Time constraint

The study was cross-sectional, and classroom observations were done at the beginning of the first term, when the teachers had been with the set of learners for less than 10 weeks. A longitudinal study would have enabled noting possible changes in their techniques of mediation after being with the set of learners for the whole year. To compensate for the reduced observation time, the in-depth interviews that were done at the end of the second and third

terms with the teachers provided room for them to explain other mediational means that they worked with but were not noted during the lessons.

6.5.2 Content delimitation

The study focused on the teachers' mediation of addition, which fell under the core element of numbers, operations, and relationships in the early years of primary school curriculum. Even though the teachers' mediation of psychomotor skills would be inferred from the way they worked with physical artefacts when carrying out the addition with the learners, it would still be interesting to see how the teachers mediated the core elements requiring more psychomotor skills—such as space, shape, and measurement.

6.5.3 Case delimitation

The selected case was a school that had a consistent record of higher learner achievement for successive years prior to the study. Even though some of the observed practices on teacher mediation were similar across the four teachers, and appeared to be traditional, the findings could not be extended to schools with other levels of learner achievement.

6.6 Suggestions for further research

The teachers selected the tasks and examples by strictly following the teachers' guide and the learners' textbooks without modifying the order to show connections between examples. This points to the need to study the affordances and limitations of the teachers' guides and learners' textbooks on highlighting variant and invariant aspects of example spaces. It could also be necessary to investigate further how the teachers' guide fails to highlight the role of artefacts as a means for supporting the progression from their physical presence to more abstract ways of working.

The teachers also mainly worked with home-made and locally available artefacts. These artefacts have their strengths and weaknesses on how they make number concepts and

relationships visible to the learners. Since the study focused on the teacher, there is need to explore the effectiveness of the traditionally used representations on learner achievement, so as to inform further development of such representations.

The participating teachers shared some common aspects of mediation regardless of their experience. For example, all the teachers used the count-all strategy for addition during all the lessons across the four classes. This points to the need to explore whether this also the case in low-performing schools—and hence establish whether the findings are applicable to typical Malawian schools.

It would also be suggested that further research should be carried out on how the teachers mediate other topics in the mathematics curriculum covering other number operations such as multiplication and division as well as other core elements of the early years mathematics curriculum. Ultimately, this points to the need to study how teacher education in Malawi and continuous professional development initiatives approach the teaching of fundamental concepts.

Since data for the study was collected during the first term of the school year, and each teacher was observed for a week, it would be worthwhile to conduct a longitudinal study of the mediational strategies used by the teachers across one or more academic years.

6.7 Personal growth

Studying the teaching of mathematics to learners in the early years of primary school has been a very interesting journey. During the initial phases of my PhD journey, I had some assumptions that did not stand the test of literature and research evidence. For instance, initially, I thought that the teaching of mathematics to young primary school children can be framed around play. Literature, however, revealed that play is only one aspect of teaching mathematics to children. My understanding of mathematics teaching became clearer after I

came across sociocultural theory upon which the MPM framework is based. It was after understanding the link between the sociocultural role of teaching and the sociocultural view of mathematics that the direction of my journey became clearer. The focus of the MPM framework fitted with the goals of my study in many ways, such as the sociocultural context of the schools that provided its empirical basis, its focus on the teaching of mathematics, and its specific focus on early years of primary school. I then proceeded on the journey, though with hurdles along the way.

The first hurdle when I started working with the usage of the MPM framework was that it had just been published by the time I had started my PhD. As such, there were no published studies exemplifying its use. The framework was developed within the Wits Maths Connect PD setup and seemed to focus on measuring differences in teaching, thereby serving the purpose of evaluating the effectiveness of the PD. The subsequent publications that exemplified the usage of the framework also seemed to lean more towards measuring the difference in the quality of mathematics teaching for teachers prior to and after their involvement in a PD programme. As such, I had to find ways of working with the framework outside this focus. Since I was not interested in evaluating the teachers in my study, but to explore their mediation strategies, I opted to only focus on the aspects of the framework that are useful for describing teaching. I came to understand how to work with the key elements of the framework through use, during the pilot study. I was then ready to collect and analyse the data for the main study.

During data analysis, I realised that the MPM framework generates a considerable volume of data for each episode. Based on the lessons from analysis of the pilot study, and one teacher from the main study, I found it necessary to learn and master qualitative data analysis software. ATLAS.ti data analysis software deemed appropriate considering its strong grip on data. This eased the analysis of the data and hence reduced the time spent when analysing each teacher. Since the lessons were taught in Chichewa, transcription and translation also required

considerable time. Unfortunately, ATLAS.ti software developers brought in useful features such as built-in transcript editing in later versions of the software while I had almost finished my data analysis.

The fundamental lesson that I have learnt while reviewing literature is that mathematics curricula used in different countries were influenced by theories of a particular historical period and share many common elements. As such, questions about the quality of the mathematics curricula have to be directed to the theoretical assumptions that informed the curriculum development, but not the teachers who are currently handling the curriculum.

The writing phase of the thesis overlapped with the global Covid-19 pandemic that brought new ways of working and interaction with colleagues and supervisors. My supervisors made it easier to adapt to these changes by being the first to adopt the new ways of working in order to maintain my progress. This has been a practical lesson for me, to put the best interests of my students even if it means working outside my comfort zone.

REFERENCES

- Adler, J. (2017). Mathematics Discourse in Instruction (MDI): A Discursive Resource as Boundary Object Across Practices. In *Proceedings of the 13th International Congress on Mathematical Education* (pp. 125–143). Springer, Cham. https://doi.org/10.1007/978-3-319-62597-3_9
- Adler, J., & Ronda, E. (2015). A framework for describing mathematics discourse in instruction and interpreting differences in teaching. *African Journal of Research in Mathematics, Science and Technology Education*, 19(3), 237–254.
- Anthony, G., & Walshaw, M. (2009). *Effective pedagogy in mathematics* (Vol. 19). International Academy of Education.
- Aploon-Zokufa, K. (2013). Locating the difference: A comparison of pedagogic strategies in high and low performing schools. *South African Journal of Childhood Education*, 3(2), 112–130.
- Aronson, E., & Bridgeman, D. (1979). Jigsaw Groups and the Desegregated Classroom: In Pursuit of Common Goals. *Personality and Social Psychology Bulletin*, 5(4), 438–446. <https://doi.org/10.1177/014616727900500405>
- Askew, M. (2012). *Transforming primary mathematics*. Routledge.
- Askew, M. (2013). Big ideas in primary mathematics: Issues and directions. *Perspectives in Education*, 31(3), 5–18.
- Askew, M. (2019). Mediating primary mathematics: Measuring the extent of teaching for connections and generality in the context of whole number arithmetic. *ZDM*, 51(1), 213–226. <https://doi.org/10.1007/s11858-018-1010-9>
- Askew, M., Venkat, H., Abdulhamid, L., Mathews, C., Morrison, S., Ramdhany, V., & Tshesane, H. (2019). *Teaching for structure and generality: Assessing changes in*

teachers mediating primary mathematics. 41–48.

<http://urn.kb.se/resolve?urn=urn:nbn:se:hj:diva-46680>

Aubrey, C., Godfrey, R., & Dahl, S. (2006). Early mathematics development and later achievement: Further evidence. *Mathematics Education Research Journal*, 18, 27–46.
<https://doi.org/10.1007/BF03217428>

Ball, D. L. (1993). With an Eye on the Mathematical Horizon: Dilemmas of Teaching Elementary School Mathematics. *The Elementary School Journal*, 93(4), 373–397.
<https://doi.org/10.1086/461730>

Ball, D. L., & Bass, H. (2002). Toward a practice-based theory of mathematical knowledge for teaching. *Proceedings of the 2002 Annual Meeting of the Canadian Mathematics Education Study Group*, 3–14.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content Knowledge for Teaching: What Makes It Special? *Journal of Teacher Education*, 59(5), 389–407.
<https://doi.org/10.1177/0022487108324554>

Bishop, A. J. (1988). Mathematics education in its cultural context. *Educational Studies in Mathematics*, 19(2), 179–191. <https://doi.org/10.1007/BF00751231>

Bishop, A. J. (2017). Elementary Mathematicians from Advanced Standpoints—A Cultural Perspective on Mathematics Education. In G. Kaiser (Ed.), *Proceedings of the 13th International Congress on Mathematical Education* (pp. 165–176). Springer International Publishing. https://doi.org/10.1007/978-3-319-62597-3_11

Boaler, J. (2016). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages and innovative teaching*. John Wiley & Sons.

Brombacher, A. (2011). *Malawi early grades mathematics assessment (EGMA): National baseline report 2010*. USAID.

https://globalreadingnetwork.net/sites/default/files/eddata/Malawi_National_Baseline_EGMA_2010.pdf

- Bussi, M. G. B., & Mariotti, M. A. (2008). Semiotic Mediation in the Mathematics Classroom Artefacts and Signs after a Vygotskian Perspective. In English, L., Bussi, M. B., Jones, G., Lesh, R., & Tirosh, D. (Eds.), *Handbook of International Research in Mathematics Education (2nd Edition)* (pp. 746–805). Mahwah: Lawrence Erlbaum.
- Charalambous, C. Y., & Praetorius, A.-K. (2018). Studying mathematics instruction through different lenses: Setting the ground for understanding instructional quality more comprehensively. *ZDM*, 50(3), 355–366. <https://doi.org/10.1007/s11858-018-0914-8>
- Chilora, H., Jessee, C., & Heyman, C. (2003). Investigating Pupils' Performance on Mathematics word problems in Lower Primary School in Malawi. Paper presented at the *Comparative and International Education Society Annual Conference*. New Orleans, LA, March 2003
- Chitera, N. (2012). Language-in-Education policies in conflict: Lessons from Malawian mathematics teacher training classrooms. *African Journal of Research in Mathematics, Science and Technology Education*, 16(1), 58–68. <https://doi.org/10.1080/10288457.2012.10740729>
- Chowdhury, M. F. (2014). Interpretivism in Aiding Our Understanding of the Contemporary Social World. *Open Journal of Philosophy*, 2014. <https://doi.org/10.4236/ojpp.2014.43047>
- Creswell, J. W. (2014). *Research Design: Qualitative, Quantitative, and Mixed Methods Approaches*. SAGE Publications.
- Daniels, H. (2017). *Introduction to Vygotsky*. Routledge.
- Davis, E. K., Bishop, A. J., & Seah, W. T. (2015). “We Don’t Understand English that is Why We Prefer English”: Primary School Students’ Preference for the Language of

- Instruction in Mathematics. *International Journal of Science and Mathematics Education*, 13(3), 583–604. <https://doi.org/10.1007/s10763-013-9490-0>
- diSessa, A. A. (2004). Metarepresentation: Native Competence and Targets for Instruction. *Cognition and Instruction*, 22(3), 293–331. https://doi.org/10.1207/s1532690xci2203_2
- Dunphy, E., Dooley, T., & Shiel, G. (2014). *Mathematics in Early Childhood and Primary Education (3–8 years:Definitions, Theories, Development and Progression*. NCCA Report 17. National Council for Curriculum and Assessment, Dublin.
- Edwards, L. D., Moore-Russo, D., & Ferrara, F. (Eds.). (2014). *Emerging Perspectives on Gesture and Embodiment in Mathematics*. Information Age Publishing (IAP).
- Ekdahl, A.-L., Venkat, H., Runesson, U., & Askew, M. (2018). Weaving in connections: Studying changes in early grades additive relations teaching. *South African Journal of Childhood Education*, 8(1), 1–9. <https://doi.org/10.4102/sajce.v8i1.540>
- Erbas, A. K., Kertil, M., Çetinkaya, B., Çakiroglu, E., Alacaci, C., & Bas, S. (2014). Mathematical Modeling in Mathematics Education: Basic Concepts and Approaches. *Educational Sciences: Theory and Practice*, 14(4), 1621–1627.
- Essien, A. A. (2018). The Role of Language in the Teaching and Learning of Early Grade Mathematics: An 11-Year Account of Research in Kenya, Malawi and South Africa. *African Journal of Research in Mathematics, Science and Technology Education*, 22(1), 48–59. <https://doi.org/10.1080/18117295.2018.1434453>
- Flyvbjerg, B. (2006). Five misunderstandings about case-study research. *Qualitative Inquiry*, 12(2), 219–245.
- Frobisher, L. (1999). *Learning to Teach Number: A Handbook for Students and Teachers in the Primary School*. Nelson Thornes.

- Goldsmith, L., & Seago, N. (2011). Using Classroom Artifacts to Focus Teachers' Noticing: Affordances and Opportunities. In M. Sherin, V. Jacobs, & P. Randy (Eds.), *Mathematics Teacher Noticing: Seeing Through Teachers' Eyes* (pp. 169–187). Routledge.
- Graven, M. (2016). When systemic interventions get in the way of localized mathematics reform. *For the Learning of Mathematics*, 36(1), 8–13.
- Graven, M., Venkat, H., Westaway, Li., & Tshesane, H. (2013). Place value without number sense: Exploring the need for mental mathematical skills assessment within the Annual National Assessments. *South African Journal of Childhood Education*, 3(2), 131–143.
- Gray, E., & Tall, D. (1993). Success and failure in mathematics: The flexible meaning of symbols as process and concept. *Mathematics Teaching*, 142(6–10).
- Gray, E., & Tall, D. (1994). Duality, Ambiguity, and Flexibility: A “Proceptual” View of Simple Arithmetic. *Journal for Research in Mathematics Education*, 25(2), 116–140.
<https://doi.org/10.2307/749505>
- Hill, H. C., Blunk, M. L., Charalambous, C. Y., Lewis, J. M., Phelps, G. C., Sleep, L., & Ball, D. L. (2008). Mathematical Knowledge for Teaching and the Mathematical Quality of Instruction: An Exploratory Study. *Cognition and Instruction*, 26(4), 430–511.
<https://doi.org/10.1080/07370000802177235>
- Hoadley, U. (2007). The reproduction of social class inequalities through mathematics pedagogies in South African primary schools. *Journal of Curriculum Studies*, 39(6), 679–706.
- Hoadley, U. (2012). What do we know about teaching and learning in South African primary schools? *Education as Change*, 16(2), 187–202.
<https://doi.org/10.1080/16823206.2012.745725>

- Hoover, M., Mosvold, R., & Fauskanger, J. (2014). Common tasks of teaching as a resource for measuring professional content knowledge internationally. *Nordic Studies in Mathematics Education*, 19(3–4), 7–20.
- Jakobsen, A., Kazima, M., & Kasoka, D. N. (2018). Assessing prospective teachers' development of MKT through their teacher education: A Malawian case. *Nordic Research in Mathematics Education*, 219.
- Jing, T. J., Tarmizi, R. A., Bakar, K. A., & Aralas, D. (2017). Utilization of variation theory in the classroom: Effect on students' algebraic achievement and motivation. *AIP Conference Proceedings*, 1795, 020028.
- Kaphesi, E. (2003). The influence of language policy in education on mathematics classroom discourse in malawi: The teachers' perspective. *Teacher Development*, 7(2), 265–285. <https://doi.org/10.1080/13664530300200190>
- Karpov, Y. V. (2003). Vygotsky's doctrine of scientific concepts. *Vygotsky's Educational Theory in Cultural Context*, 65–82.
- Kasoka, D. N., Jakobsen, A., & Kazima, M. (2017, February). Preparing teachers for teaching: Does initial teacher education improve mathematical knowledge for teaching? *CERME 10*. <https://hal.archives-ouvertes.fr/hal-01949148>
- Katzin, N., Cohen, Z. Z., & Henik, A. (2019). If it looks, sounds, or feels like subitizing, is it subitizing? A modulated definition of subitizing. *Psychonomic Bulletin & Review*, 26(3), 790–797. <https://doi.org/10.3758/s13423-018-1556-0>
- Kaur, B. (2017). Mathematics Classroom Studies: Multiple Lenses and Perspectives. In G. Kaiser (Ed.), *Proceedings of the 13th International Congress on Mathematical Education* (pp. 45–61). Springer International Publishing.
- Kazima, M. (2008). Mother tongue policies and mathematical terminology in the teaching of mathematics. *Pythagoras*, 2008(1), 53–63.

- Kazima, M., & Adler, J. (2006). Mathematical knowledge for teaching: Adding to the description through a study of probability in practice. *Pythagoras*, 2006(63), 46–59.
- Kazima, M., Jakobsen, A., & Kasoka, D. N. (2016). Use of Mathematical Tasks of Teaching and the Corresponding LMT Measures in the Malawi Context. *The Mathematics Enthusiast*, 13(1), 171–186.
- Kilpatrick, J., Swafford, J., & Findell, B. (2001). The strands of mathematical proficiency. *Adding It up: Helping Children Learn Mathematics*, 115–118.
- Kozulin, A. (2003). Psychological tools and mediated learning. *Vygotsky's Educational Theory in Cultural Context*, 15–38. <https://doi.org/10.1017/CBO9780511840975.003>
- Kullberg, A., Kempe, U. R., & Marton, F. (2017). What is made possible to learn when using the variation theory of learning in teaching mathematics? *ZDM*, 49(4), 559–569. <https://doi.org/10.1007/s11858-017-0858-4>
- Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal*, 27(1), 29–63.
- Leung, A. Y.-L. (2012). Variation and mathematics pedagogy. *The 35th Annual Conference of the Mathematics Education Research Group of Australasia (MERGA)*, 433–440.
- Malawi Institute of Education. (2012a). *Masamu: Buku la Ophunzira la Standade 1*. Malawi Institute of Education (MIE).
- Malawi Institute of Education. (2012b). *Mathematics Teachers' Guide for Standard 1*. Malawi Institute of Education (MIE).
- Malawi Institute of Education. (2012c). *Numeracy and Mathematics Teachers' Guide for Standard 2*. Malawi Institute of Education (MIE).
- Malawi Institute of Education. (2013a). *Mathematics Teachers' Guide for Standard 3*. Malawi Institute of Education (MIE).

- Malawi Institute of Education. (2013b). *Mathematics Teachers' Guide for Standard 4*. Malawi Institute of Education (MIE).
- Marton, F., & Booth, S. A. (1997). *Learning and awareness*. Routledge.
- Marton, F., Tsui, A. B., Chik, P. P., Ko, P. Y., & Lo, M. L. (2004). *Classroom discourse and the space of learning*. Routledge.
- Mhlolo, M. (2013). The merits of teaching mathematics with variation: Original research. *Pythagoras*, 34(2), 1–8. <https://doi.org/10.4102/pythagoras.v34i2.233>
- Mok, I. A. C. (2017). Teaching Algebra through Variations. In *Teaching and Learning Mathematics through Variation* (pp. 187–205). Springer.
- Mosvold, R. (2008). Real-life connections in Japan and the Netherlands: National teaching patterns and cultural beliefs. *International Journal for Mathematics Teaching and Learning*.
- Mosvold, R. (2016). The work of teaching mathematics from a commognitive perspective. In W. Mwakapenda, T. Sedumedi, & M. Makgato (Eds.), *Proceedings of the 24th annual conference of the Southern African Association for Research in Mathematics, Science and Technology Education (SAARMSTE)* (pp. 186–195). SAARMSTE.
- Mosvold, R., & Fauskanger, J. (2018). Opportunities and challenges of using the MDI framework for research in Norwegian teacher education. *Nordic Research in Mathematics Education*, 209.
- Mtika, P., & Gates, P. (2010). Developing learner-centred education among secondary trainee teachers in Malawi: The dilemma of appropriation and application. *International Journal of Educational Development*, 30, 396–404. <https://doi.org/10.1016/j.ijedudev.2009.12.004>
- Muir, T. (2008). Zero is not a number: Teachable moments and their role in effective teaching of numeracy. *MERGA*, 2, 361–367.

- Ng, D., Mosvold, R., & Fauskanger, J. (2012). Translating and Adapting the Mathematical Knowledge for Teaching (MKT) Measures: The Cases of Indonesia and Norway. *The Mathematics Enthusiast*, 9(1), 149–178.
- Nguyen, N. T., McFadden, A., Tangen, D. D., & Beutel, D. D. (2013). *Video-stimulated recall interviews in qualitative research*. 10.
- Niss, M. (2012). Models and modelling in mathematics education. *Ems Newsletter*, 86, 49–52.
- Palyst, T. (2008). Purposive Sampling. In *The Sage Encyclopedia of Qualitative Research Methods*. Sage Publications.
- Parker, A., & Faulkner, J. (2004). *Excel Basic Skills Mental Maths Strategies: Year 5*. Pascal Press.
- Piaget, J. (1953). *The origin of intelligence in the child*. Routledge & Kogan Paul.
- Ravishankar, V., El-Kogali, S. E.-T., Sankar, D., Tanaka, N., & Rakoto-Tiana, N. (2016). *Primary Education in Malawi: Expenditures, Service Delivery, and Outcomes*. World Bank Publications.
- Rezat, S., & Sträßer, R. (2012). From the didactical triangle to the socio-didactical tetrahedron: Artifacts as fundamental constituents of the didactical situation. *ZDM*, 44(5), 641–651. <https://doi.org/10.1007/s11858-012-0448-4>
- Richmond, J. E., & Taylor, M. (2014). Visual recognition difficulties: Identifying primary school learners' directional confusion in writing letters and numbers. *South African Journal of Occupational Therapy*, 44(3), 2–6.
- Rittle-Johnson, B., Fyfe, E. R., Hofer, K. G., & Farran, D. C. (2017). Early Math Trajectories: Low-Income Children's Mathematics Knowledge From Ages 4 to 11. *Child Development*, 88(5), 1727–1742. <https://doi.org/10.1111/cdev.12662>
- Roberts, N. (2016). Additive Relations Word Problems in the South African Curriculum and Assessment Policy Standard at Foundation Phase. *African Journal of Research in*

- Mathematics, Science and Technology Education*, 20(2), 106–118.
<https://doi.org/10.1080/18117295.2016.1189212>
- Robertson, S., Cassity, E., & Kunkwenzu, E. (2017). *Girls' Primary and Secondary Education in Malawi: Sector Review*.
- Roth, W.-M., & Lee, Y.-J. (2007). "Vygotsky's neglected legacy": Cultural-historical activity theory. *Review of Educational Research*, 77(2), 186–232.
- Rouleau, A., & Liljedahl, P. (2017). Teacher Tensions: The Case of Naomi. In C. Andrà, D. Brunetto, E. Levenson, & P. Liljedahl (Eds.), *Teaching and Learning in Maths Classrooms: Emerging Themes in Affect-related Research: Teachers' Beliefs, Students' Engagement and Social Interaction* (pp. 155–162). Springer International Publishing.
https://doi.org/10.1007/978-3-319-49232-2_15
- Rowland, T., Turner, F., Thwaites, A., & Huckstep, P. (2009). *Developing Primary Mathematics Teaching: Reflecting on Practice with the Knowledge Quartet*. SAGE Publications.
- Ruiz, C., & Balbi, A. (2019). The effects of teaching mental calculation in the development of mathematical abilities. *The Journal of Educational Research*, 112(3), 315–326.
<https://doi.org/10.1080/00220671.2018.1519689>
- Saka, T. W. (2019). *An Exploration of Mathematics Classroom Culture in Selected Early Grade Mathematics Classrooms in Malawi*. University of Johannesburg.
- Saka, T. W., & Roberts, N. (2018). Manipulatives for early grade whole number and relationships: The potential of the Malawian bow-abacus. *Bloemfontein, South Africa*.
- Saunders, M. N. K., Lewis, P., & Thornhill, A. (2009). *Research methods for business students* (5th ed). Prentice Hall.

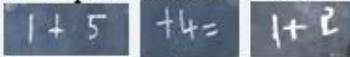
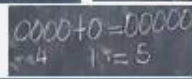
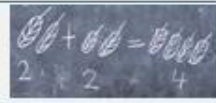
- Schifter, D. (2011). Examining the Behavior of Operations: Noticing Early Algebraic Ideas. In M. Sherin, V. Jacobs, & R. Philipp (Eds.), *Mathematics Teacher Noticing: Seeing Through Teachers' Eyes*, (pp. 66-78). Routledge.
- Schoenfeld, A. (2007). Method. In F. K. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning: A Project of the National Council of Teachers of Mathematics*, (pp. 69-107). IAP.
- Sfard, A. (2007). When the rules of discourse change, but nobody tells you: Making sense of mathematics learning from a commognitive standpoint. *The Journal of the Learning Sciences*, 16(4), 565–613.
- Sfard, A. (2008). *Thinking as communicating: Human development, the growth of discourses, and mathematizing*. Cambridge university press.
- Shapiro, L. (2019). *Embodied Cognition*. Routledge.
- Skemp, R. R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching*, 77(1), 20–26.
- Speiser, B., & Walter, C. (2011). Models for products. *The Journal of Mathematical Behavior*, 30(4), 271–290. <https://doi.org/10.1016/j.jmathb.2011.05.001>
- Stohlmann, M. S., & Albarracín, L. (2016, June 28). *What Is Known about Elementary Grades Mathematical Modelling* [Review Article]. Education Research International; Hindawi. <https://doi.org/10.1155/2016/5240683>
- Tabulawa, R. (2013). *Teaching and learning in context: Why pedagogical reforms fail in Sub-Saharan Africa*. African Books Collective.
- TIMSS. (2015). *Student Achievement – TIMSS 2015 and TIMSS Advanced 2015 International Results*. <http://timssandpirls.bc.edu/timss2015/international-results/timss-2015/mathematics/student-achievement/>

- Tindana, P. O., Kass, N., & Akweongo, P. (2006). The Informed Consent Process in a Rural African Setting: *IRB*, 28(3), 1–6.
- Venkat, H., & Askew, M. (2018). Mediating primary mathematics: Theory, concepts, and a framework for studying practice. *Educational Studies in Mathematics*, 97(1), 71–92. <https://doi.org/10.1007/s10649-017-9776-1>
- Walshaw, M., & Anthony, G. (2008). Creating Productive Learning Communities in the Mathematics Classroom: An International Literature Review. *Pedagogies: An International Journal*, 3(3), 133–149. <https://doi.org/10.1080/15544800802026595>
- Wanat, C. L. (2008). Getting Past the Gatekeepers: Differences Between Access and Cooperation in Public School Research. *Field Methods*, 20(2), 191–208. <https://doi.org/10.1177/1525822X07313811>
- Wasunna, C., Tegli, J., & Ndebele, P. (2014). Informed consent in an African context. In M. Kruger, P. Ndebele and L. Horn (Eds.), *Research Ethics in Africa: A Resource for Research Ethics Committees*, (pp. 57–62). SUN MeDIA Stellenbosch.
- Watts, T. W., Duncan, G. J., Clements, D. H., & Sarama, J. (2018). What Is the Long-Run Impact of Learning Mathematics During Preschool? *Child Development*, 89(2), 539–555. <https://doi.org/10.1111/cdev.12713>
- Wertsch, J. V. (2017, April 26). Mediation. In H. Daniels (Ed.), *Introduction to Vygotsky* (pp. 59-72). Routledge. <https://doi.org/10.4324/9781315647654-3>
- Wilson, M. (2002). Six views of embodied cognition. *Psychonomic Bulletin & Review*, 9(4), 625–636. <https://doi.org/10.3758/BF03196322>
- Wilson, R., A., & Foglia, L. (2017). *Embodied Cognition*. <https://plato.stanford.edu/archives/spr2017/entries/embodied-cognition/>
- Wright, R. J., & Ellemor-Collins, D. (2018). *The Learning Framework in Number: Pedagogical Tools for Assessment and Instruction*. SAGE Publications.

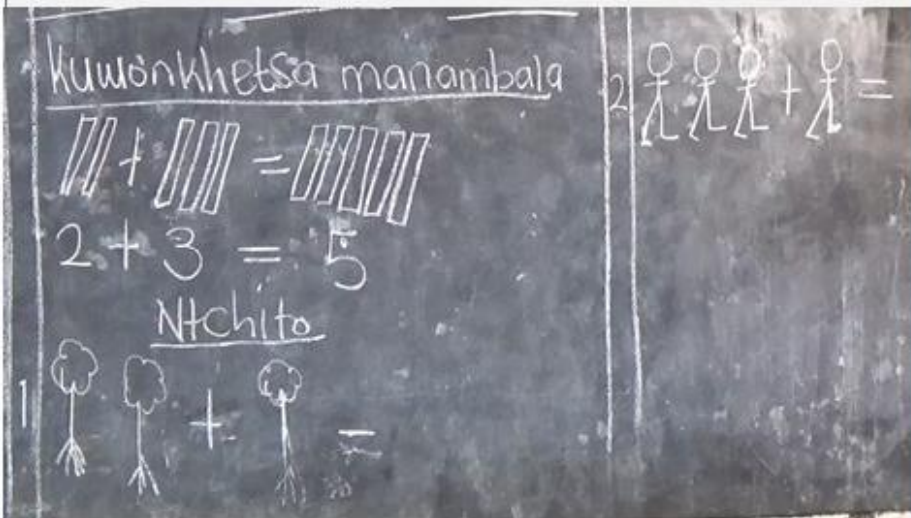
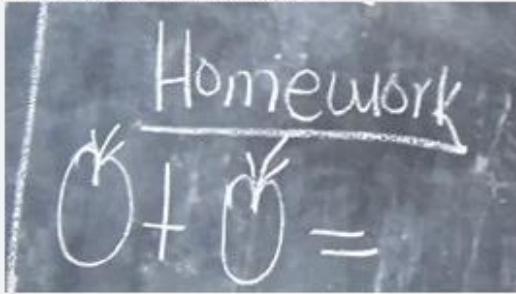
- Wright, R. J., Stanger, G., Stafford, A. K., & Martland, J. (2014). *Teaching Number in the Classroom with 4-8 Year Olds*. SAGE.
- Yin, R. K. (2016). *Qualitative research from start to finish (2nd.)*. New York, NY: The Guilford Press.

APPENDICES

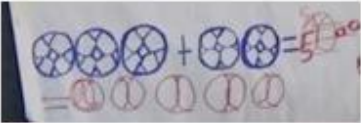
Appendix 1: Lesson graph for Lesson 1 of Standard 1

LESSON GRAPH FOR STANDARD 1 LESSON 1	
2¾ min (Whole class)	Teacher explains the meaning of addition.
6¾ min (Whole class)	Introduces the + sign. Asks learners to be saying “Dot! Down! Cut-in-the-middle!” as they write the sign in the air. Writes the = sign on the chalkboard. Presented the verbalised hand movement as “Dot! To-the-right! And Dot! To-the-right!”.
7½ min (Group work)	Asks learners to add two books and 1 book in their groups. Presents the result using the statement “2 plus 1 equals 3”; saying it together with the class.
8¼ min (Whole class)	Asks learners to take turns trying out writing the statement: “2 plus 1 equals 3” on the chalkboard. The 10th learner wrote it as expected. 
3½ min (Group work)	Asks learners to add 4 stones and 1 stone in their groups and presents the result using the statement “4 plus 1 equals 5”.
5 min (Whole class)	Ask learners to try out writing the statement: “4 plus 1 equals 5” on the chalkboard.  The teacher took over and sketched the stones with their corresponding numbers. 
4½ min (Group work)	Asks learners to add 2 leaves and other 2 leaves in their groups. Presents the result using the statement “2 plus 2 equals 4”.
3 min (Whole class)	Draws leaves on the chalkboard and writes the numbers below the groups of leaves. 
LESSON TIME: 41 MINUTES	

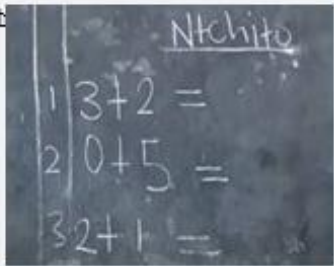
Appendix 2: Lesson graph for Lesson 2 of Standard 1

LESSON GRAPH FOR STANDARD 1 LESSON 2	
4 min (Whole class)	Reviews + and = signs from the morning lesson.
7 min (Group work)	Asks learners in their groups to pick 2 sticks followed by 3 sticks and find the sum of the picked sticks. Draws the sticks on the chalkboard.
6½ min (Whole class)	Asks learners to take turns trying to write the statement $2 + 3 = 5$ on the chalkboard.
4½ min (whole class)	Asks learners to quickly tell $2 + 1$ and $3 + 0$ mentally.
60 min (Individual work)	Asks learners to solve $2 + 1$ and $3 + 1$ presented using drawings:  Gives learners homework: 
LESSON TIME: 1 hr 22 MINUTES	

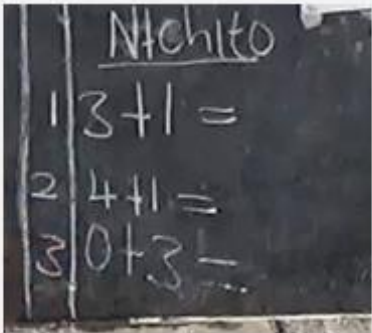
Appendix 3: Lesson graph for Lesson 3 of Standard 1

LESSON GRAPH FOR STANDARD 1 LESSON 3	
4 min <i>(Whole class)</i>	Reviews + and = signs from the previous day's lesson.
17½ min <i>(Whole class)</i>	Asks learners to take turns trying out writing the statement verbally presented as : "2 leaves plus 1 leaf equals 3 leaves" on the chalkboard. The 4th learner wrote it as expected. 
20 min <i>(Whole class)</i>	Teacher visually presents $3 + 2 =$ as a drawing on a chart. Four learners try out giving the answer, but they fail to present it as required by the teacher.  Teacher gives the expected answer: 
14¼ min <i>(Whole class followed by individual work)</i>	Presents two problems on a chart. Read the first problem with the class and asks learners to write down the answer. After one learners fails to present the answer as required by the teacher, all learners are asked to individually write the problem in their notebooks, and marks. 
LESSON TIME: 56 MINUTES	

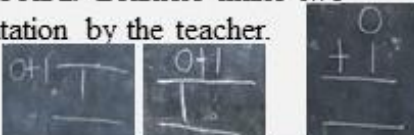
Appendix 4: Lesson graph for Lesson 4 of Standard 1

LESSON GRAPH FOR STANDARD 1 LESSON 4	
9¼ min <i>(Whole class)</i>	Reviews previous learning by first asking learners to write the + sign. Thereafter, asks learners to present the statement “2 balls plus 2 balls equals” on the chalkboard. Ends by asking learners to find the answer.
11½ min <i>(Whole class)</i>	Asks learners to write the solution for $2 + 0 =$ posted on the chalkboard. Learners take turns writing 0, 2, and 3 as answers. Teacher works out the correct answer with the class using counters.
7¼ min <i>Group work</i>	Distributes papers with pre-written addition statements ($2 + 1$, $3 + 1$, $4 + 1$, $1 + 1$, $5 + 0$, $1 + 2$, $3 + 0$, $2 + 2$, $2 + 3$, $4 + 0$) to be solved in groups.
23¼ min <i>(Whole class)</i>	Group representatives post the papers on the chalkboard. Teacher and class use counters to verify the solutions given by the groups. Teacher and class verifies $2 + 1$, $3 + 1$, $4 + 1$, $1 + 1$, $5 + 0$, $1 + 2$ and $3 + 0$. 
31¼ min <i>(Whole class followed by individual work)</i>	Presents three problems ($3 + 2 =$, $0 + 5 =$ and $2 + 1$) on the chalkboard. Asks learners to write the problems in their notebooks and starts marking. Asks learners to go outside for a break, to continue work after break. 
LESSON TIME: 1 HOUR 23 MINUTES	

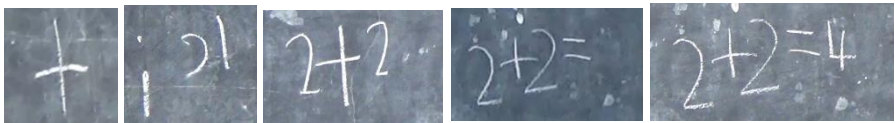
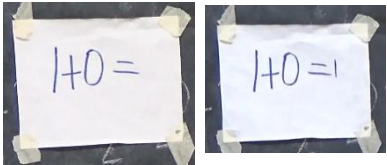
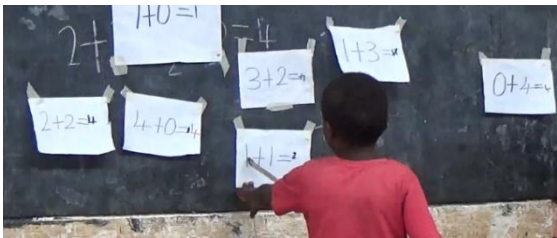
Appendix 5: Lesson graph for Lesson 5 of Standard 1

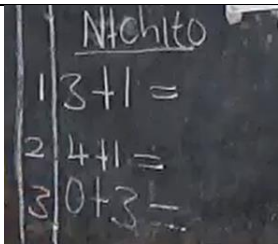
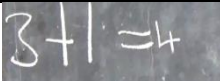
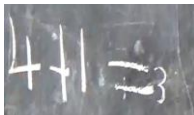
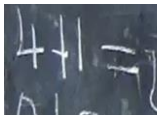
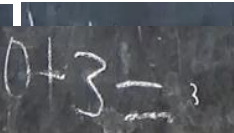
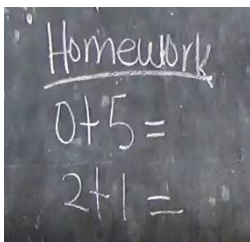
LESSON GRAPH FOR STANDARD 1 LESSON 5	
8 min (Whole class)	Reviews previous learning by first asking learners to state what they have been learning. Learners finally came up with $2 + 2 = 4$ as an example of an addition statement.
5¼ min (Whole class)	Asks learners to solve $1 + 0 =$ posted on the chalkboard. One learner writes 1 as the answer, later verified with the class using counters.
5¼ min (Group work)	Distributes papers with addition statements to be solved in groups ($2 + 2$, $4 + 0$, $1 + 1$, $3 + 2$, $1 + 3$, $0 + 4$, $2 + 1$, and $2 + 0$).
17¾ min (Whole class)	Group representatives post their papers with on the chalkboard. Teacher and class use counters to verify the solutions given by the groups. Teacher and class verifies $2 + 2$, $4 + 0$, $1 + 1$, $3 + 2$, $1 + 3$, $0 + 4$, $2 + 1$, and $2 + 0$. 
33½ min (Individual work)	Presents three problems on the chalkboard: $3 + 1 =$, $4 + 1 =$, and $0 + 3 =$. Asks learners to write the problems in their notebooks and marks the learners' work. 
17½ min (Whole class)	Asks learners to write the answers to the given problems on the chalkboard. The teacher verifies the offered solutions with the class using counters. Gives two problems ($0 + 5 =$ and $2 + 1 =$) as homework. 
LESSON TIME: 1 HOUR 27 MINUTES	

Appendix 6: Lesson graph for Lesson 6 of Standard 1

LESSON GRAPH FOR STANDARD 1 LESSON 6	
6½ min <i>(Whole class)</i>	Reviews the + sign and = sign with the class, asking learners to write.
11¼ min <i>(Whole class)</i>	Introduces vertical addition asking learners to take turns writing the statement “0 plus 1 equals” on the chalkboard. Learners make two attempts, followed by the correct presentation by the teacher. 
11¾ min <i>(Whole class)</i>	$\begin{array}{r} 1 \\ + 3 \\ \hline \end{array}$ Asks learners to solve $\begin{array}{r} 1 \\ + 3 \\ \hline \end{array}$ posted on the chalkboard. Four learners successively write the answers as 5, 3, 4 and 4. Teacher and class verify the correct answer using counters.
4½ min <i>(Whole class)</i>	Distributes papers with vertical addition statements to be solved in groups (1 + 1, 4 + 0, 2 + 2, 3 + 1, 5 + 0, 3 + 2, 1 + 2, 1 + 3, 0 + 1, 0 + 2, 2 + 1)
25 min <i>(Whole class)</i>	Group representatives post their papers with on the chalkboard. Teacher and class use counters to verify the solutions given by the groups. 
24¾ min <i>(Individual work)</i>	$\begin{array}{r} 1 \\ + 4 \\ \hline \end{array}$ $\begin{array}{r} 2 \\ + 0 \\ \hline \end{array}$ $\begin{array}{r} 3 \\ + 1 \\ \hline \end{array}$ Presents three problems on the chalkboard: $\begin{array}{r} 1 \\ + 4 \\ \hline \end{array}$, $\begin{array}{r} 2 \\ + 0 \\ \hline \end{array}$, and $\begin{array}{r} 3 \\ + 1 \\ \hline \end{array}$ Asks learners to write the problems in their notebooks and marks the learners’ work. 
LESSON TIME: 1 HOUR 25 MINUTES	

Appendix 7: Episode summaries for Lesson 5 of Standard 1

STANDARD 1 LESSON 5 EPISODE SUMMARIES		
Episode	Task	Episode summary
1	Review of previous learning (Teacher and whole class)	Asks learners to explain what they are learning in mathematics. One learner said they were adding, while another one used a redundant phrase sounding like “adding the plus”. Asks learners to write an addition sentence on the chalkboard. One learner writes the + sign, the second learner writes some unknown characters looking like “i i”. The third learner wrote $2 + 2$. After asking the class to read “$2 + 2$” the learners felt that it was incomplete; and the fourth learner wrote it as $2 + 2 =$. The last learner appended the answer, 4, after the equal sign.  The teacher explained the meaning of an addition sentence as the statement that presents the question for the learners to add, giving an example of $2 + 2 =$
2	Finding $1 + 0$ (Teacher and whole class)	Asks learners to read the statement $1 + 0 =$ on a piece of paper posted on the chalkboard. Emphasises to learners to always point at the numbers when reading them while standing in the front close to the chalkboard. Asks learners to write the answer to the given statement. One learner writes 1. Asks the class to read the entire statement and unanimously agree that the just written answer is correct.  Asks all learners to pick their counters to check whether the given answer is really correct.
3	3.1	Finding $2 + 2, 4 + 0, 1 + 1, 3 + 2, 1 + 3, 0 + 4, 2 + 1, 2 + 0$ (Groupwork) Distributes papers with prewritten addition statements ($2 + 2, 4 + 0, 1 + 1, 3 + 2, 1 + 3, 0 + 4, 2 + 1, 2 + 0$) to groups.  Asks learners to work together to find the answer to their given problem and write it down.
	3.2	Verifying solutions for $2 + 2, 4 + 0, 1 + 1, 3 + 2, 1 + 3, 0 + 4, 2 + 1, 2 + 0$ (Teacher and whole class) Asks group representatives to stick the papers with their given statements on the chalkboard.  Asks group representatives to remain in front and wait for their turn to present their solution to the class. The teacher works with the class to verify the solutions given by the groups using counters.

4	4.1	Finding $3 + 1$, $4 + 1$, and $0 + 3$ (<i>Individual work</i>)	Writes $3 + 1$, $4 + 1$, and $0 + 3$ on the chalkboard for learners to write in their notebooks and reminds them to always write the main topic (<i>Addition of numbers up to 5</i>). Marks the learners' work and offers individual help where necessary.	
	4.2	Verifying solutions for $3 + 1$, $4 + 1$, and $0 + 3$ (<i>Teacher and whole class</i>)	Asks learners to come forward and write the answer for each of the problems. Asks the class to verify the offered answers for each of the three problems using counters. Asks the learners to copy $0 + 5 =$ and $2 + 1 =$ written on the chalkboard as their homework.	    
TOTAL LESSON TIME: 1 HOUR 27 MINUTES				

Appendix 8: Lesson graph for Lesson 1 of Standard 2

LESSON GRAPH FOR STANDARD 2 LESSON 1	
4½ min (Whole class)	Teacher vertically writes $12 + 5 =$ on the board and solves it with the whole class using column addition strategies. Teacher introduces the days topic.
2¼ min (Whole class)	Teacher horizontally writes $1 + 9 =$ and solves it with the whole class.
1¼ min (Whole class)	Teacher horizontally writes $6 + 4 =$ and solves it with the whole class.
3 min (Two learners)	Teacher horizontally writes $2 + 8 =$ and $5 + 5 =$ and asks two learners to come in front and solve.
2 min (Whole class)	Teacher and class verify the answers given by the two learners.
5 ½ min (Group work)	The teacher distributes single sheets of paper to 8 groups of learners, each with all the three addition statements: $8 + 2 =$, $3 + 7 =$ and $5 + 5 =$. Asks representatives to line up in front and display their answers.
2¼ min (Whole class)	Teacher with the entire class verifies all the answers from the groups
16¼ min (Individual work)	Teacher writes three problems: $11 + 6 =$, $14 + 5 =$, and $15 + 3 =$ on the chalkboard. Asks learners to solve independently in their notebooks.
7 min (Whole class)	Teacher works with learners to find the solutions to the problems given in the preceding individual activity. Gives a brief summary of the lesson
TOTAL LESSON TIME: 41 MINUTES	

Appendix 9: Lesson graph for Lesson 2 of Standard 2

LESSON GRAPH FOR STANDARD 2 LESSON 2	
2½ min (Whole class)	Review of previous lesson (conducted two days prior to this lesson).
3¾ min (Whole class)	Teacher horizontally writes $3 + 9 =$ and solves it with the whole class.
3½ min (Two learners)	Teacher horizontally writes $4 + 8 =$ and $5 + 7 =$ and asks two learners to come in front and solve.
2¼ min (Whole class)	Teacher and class verify the answers given by the two learners.
5¾ min (Group work)	The teacher distributes single sheets of paper to 9 groups of learners, each with two addition statements: $9 + 3 =$ and $8 + 4 =$. Asks representatives to line up in front and display their answers.
2¼ min (Whole class)	Teacher with the entire class verifies all the answers from the groups
16¼ min (Individual work)	Teacher writes three problems: $6 + 6 =$ and $7 + 5 =$ on the chalkboard. Asks learners to solve independently in their notebooks.
3 min (Whole class)	Teacher works with learners to find the solutions to the problems given in the preceding individual activity and ends the lesson.
TOTAL LESSON TIME: 44 MINUTES	

Appendix 10: Lesson graph for Lesson 3 of Standard 2

LESSON GRAPH FOR STANDARD 2 LESSON 3 Lesson 3 was observed 1 month after lessons 1 and 2	
5 min (Whole class)	Teacher reviews previous lesson on counting from 1 to 50. Invites 4 learners in turn.
14½ min (Whole class)	Teacher writes 35 and 13 on separate sections of the chalkboard. Teacher asks learners to re-write 35 and 13 under place-value headings for tens and ones (T and O). Teacher uses bundles of sticks to represent tens, and loose sticks to represent ones in two place-value boxes marked with T and O, for 35 and 13, then finds the sum 48. Verifies the answer using column addition. 
5 min (Learner in front)	A learner uses two place-value boxes to find $28 + 11$
4½ min (Whole class)	Teacher verifies the answer for $28 + 11$ with the whole class using place-value boxes and very quick column addition.
10 min (Whole class)	Learners take turns to represent 45, 2 and $45 + 2$ using place-value boxes. Teacher explains what the learners did, and verifies the answer with counters using column addition.
17 ½ min (Individual work)	Teacher writes two problems: $6 + 22$ and $36 + 10$ on the chalkboard. Learners solve independently in their notebooks and the teacher goes around to mark.
6 min (Whole class)	Teacher with the whole class to find the solutions for $6 + 22$ and $36 + 10$ from the individual activity using place-value boxes
TOTAL LESSON TIME: 1 HOUR 3 MINUTES	

Activity 3 Addition facts

Time allocation: 4 lessons

Suggested teaching and learning resources

You will need the following resources:

- counters
- number cards
- charts

Instructions

- 1 Ensure that the learners are in groups and that each group has counters.

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- 2 Display a chart showing incomplete basic addition facts of 10 and 12.
- 3 Distribute cards containing incomplete addition facts of any number between 1 and 12.

Basic addition facts of 10

$$\begin{array}{rcl} 1 + 9 & = & \\ 9 + 1 & = & \\ 2 + 8 & = & \\ 8 + 2 & = & \\ 3 + 7 & = & \\ 7 + 3 & = & \\ 5 + 5 & = & \\ 4 + 6 & = & \\ 6 + 4 & = & \end{array}$$

Basic addition facts of 12

$$\begin{array}{rcl} 3 + 9 & = & \\ 9 + 3 & = & \\ 4 + 8 & = & \\ 8 + 4 & = & \\ 6 + 6 & = & \\ 7 + 5 & = & \\ 5 + 7 & = & \end{array}$$

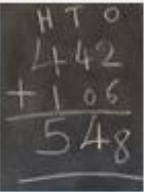
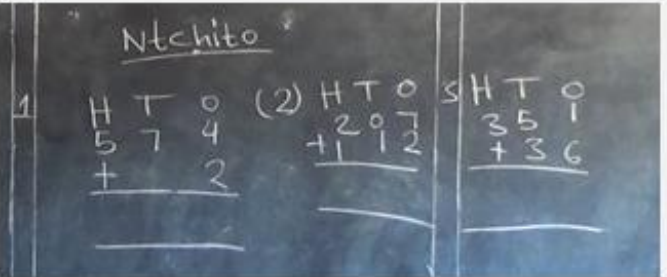
- 4 Let the learners complete the addition facts using counters.
- 5 Ask the learners to report their findings for 9 class discussion.
- 6 Help the learners to master addition facts using various ways such as mental sums.
- 7 Let the learners turn to *Mutu* 2 of their books and do *Ntchito* 3 individually.

Ntchito 3 Kubwereza kuwonkhetsa nambala mpaka 20

Wonkhetsani nambala zotsatirazi.

1	11 + 6 =	5	15 + 3 =
2	14 + 5 =	6	16 + 1 =
3	17 + 2 =	7	8 + 7 =
4	9 + 11 =	8	13 + 0 =
9	$\begin{array}{r} 10 \\ + 9 \\ \hline \hline \end{array}$	13	$\begin{array}{r} 7 \\ + 10 \\ \hline \hline \end{array}$
10	$\begin{array}{r} 12 \\ + 5 \\ \hline \hline \end{array}$	14	$\begin{array}{r} 8 \\ + 12 \\ \hline \hline \end{array}$
11	$\begin{array}{r} 16 \\ + 4 \\ \hline \hline \end{array}$	15	$\begin{array}{r} 6 \\ + 13 \\ \hline \hline \end{array}$
12	$\begin{array}{r} 13 \\ + 3 \\ \hline \hline \end{array}$	16	$\begin{array}{r} 8 \\ + 4 \\ \hline \hline \end{array}$

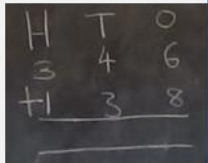
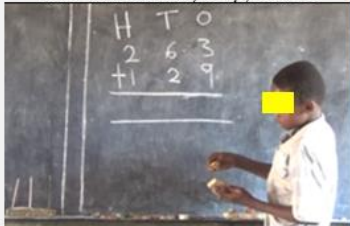
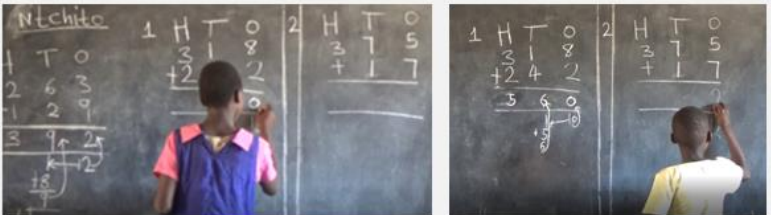
Appendix 13: Lesson graph for Lesson 1 of Standard 3

LESSON GRAPH FOR STANDARD 3 LESSON 1	
2¼ min (Whole class)	Reviews skip counting in 10's from 300 to 600.
24 min (Group work)	Introduces the addition of 3-digit numbers using an abacus. Writes $442 + 106$ in column layout on the chalkboard. Asks learners to read each addend. Distributes a pair of hand-made spiked abaci marked H T O to groups. Asks learners to stack counters (bottle-tops) of the two abaci corresponding to the given addends. Asks learners to add counters of the second abacus, representing the second addend to the first abacus, with respect to the corresponding place-value of each spike. Writes the sum of the counters on the chalkboard as they are added on each spike. Goes round the classroom to check if the learners placed the counters on the abaci as expected.    
11¾ min (Individual work)	Asks learners to solve $574 + 2$, $207 + 112$, and $351 + 36$ written on the chalkboard. Starts marking. Collects unmarked notebooks. 
9½ min (Learners in front and the whole class)	Verifies the solutions to the individual work with the class. Works out $574 + 2$ with the class using counters Successively asks two learners to lead working out $207 + 112$ and $351 + 36$ together with the class. Ends the lesson.  
LESSON TIME: 47 MINUTES	

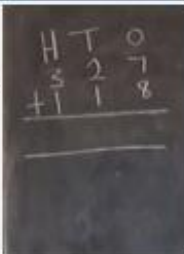
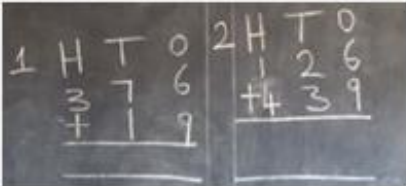
Appendix 14: Lesson graph for Lesson 2 of Standard 3

LESSON GRAPH FOR STANDARD 3 LESSON 2	
2½ min <i>(Whole class)</i>	Reviews skip counting in 10's from 400 to 600.
14 min <i>(Whole class)</i>	Writes $541 + 27$ in column layout on the chalkboard under the place-value headings H T O . Asks a learner to read the given problem. Assigns the addends to each of two abaci. Works with the class to add the corresponding sets of counters representing the second addend to the first abacus, with respect to the corresponding place-value of each spike. Writes the sum of the counters on the chalkboard as they are added on each spike. 
8¼ min <i>(Group work)</i>	Writes $412 + 167$ in column layout on the chalkboard under the place-value headings H T O . Asks learners to work out the solution in their groups. The teacher works with the groups in the process leading to the required answer. 
11¼ min <i>(Individual work)</i>	Asks learners to solve $425 + 42$ and $361 + 128$ written on the chalkboard. Starts marking. Collects unmarked notebooks. 
11¾ min <i>(Learners in front and the whole class)</i>	Verifies the solutions to the individual work with the class. Successively asks two learners to lead the class in working out the solutions to $425 + 42$ and $361 + 128$ using abaci.  Ends the lesson.
LESSON TIME: 48 MINUTES	

Appendix 15: Lesson graph for Lesson 3 of Standard 3

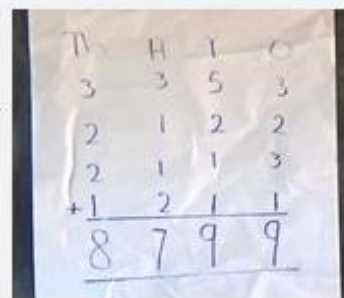
LESSON GRAPH FOR STANDARD 3 LESSON 3	
3¾ min (Whole class)	Asks the class to skip count in 10's from 450 to 600.
19¼ min (Whole class)	Writes $346 + 138$ under the place-value headings H T O. Assigns the addends to two abaci. Records the regrouping procedure on the chalkboard as the corresponding counters from the two abaci are added. 
9¼ min (Group work)	Writes $263 + 129$ in column layout on the chalkboard under the place-value headings H T O. The teacher works with learners in groups in the process leading to the required answer. 
19¼ min (A learner in front and the whole class)	Asks a learner to work out the solution to $263 + 129$ in front, together with the class, using a pair of abaci. The learner records the procedure on the chalkboard. The teacher repeats the process with the whole class and rewrites the answer found by the learner on the chalkboard. 
13¼ min (Individual work)	Asks learners to solve $318 + 242$ and $375 + 17$ written on the chalkboard. Starts marking. Collects unmarked notebooks. 
18¼ min (Learners in front and the whole class)	Successively asks two learners to lead the class in working out the solutions to $318 + 242$ and $375 + 17$ using abaci.  Ends the lesson
LESSON TIME: 1 HOUR 23 MINUTES	

Appendix 16: Lesson graph for Lesson 4 of Standard 3

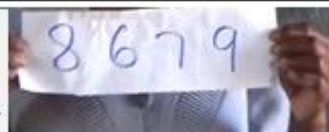
LESSON GRAPH FOR STANDARD 3 LESSON 4	
6 min <i>(Whole class)</i>	Asks the class to skip count in 10's from 500 to 600. Asks some learners to skip-count in 5's from 500 to 600.
12¾ min <i>(Whole class)</i>	Writes $327 + 118$ under the place-value headings H T O. Assigns the addends to two abaci. Records the regrouping procedure on the chalkboard as the corresponding counters from the two abaci are added. 
9¾ min <i>(Group work)</i>	Writes $519 + 6$ in column layout on the chalkboard under the place-value headings H T O. Asks learners to work out the solution in their groups. Works with the groups in the process leading to the required answer using a pair of abaci. 
8½ min <i>(A learner in front and the whole class)</i>	Asks a learner to work out the solution to $519 + 6$ in front, together with the class, using a pair of abaci. The teacher repeats the process with the whole class and rewrites the answer found by the learner. 
9¾ min <i>(Individual work)</i>	Asks learners to solve $376 + 19$ and $126 + 439$ written on the chalkboard. Starts marking. Collects unmarked notebooks. 
18 min <i>(Whole class)</i>	Verifies the solutions to the individual work with the class. Asks a learner to lead the class in working out the solutions to $376 + 19$ followed by the teacher solving $126 + 439$ using abaci. Ends the lesson. 
LESSON TIME: 56 MINUTES	

Appendix 17: Lesson graph for Lesson 1 of Standard 4

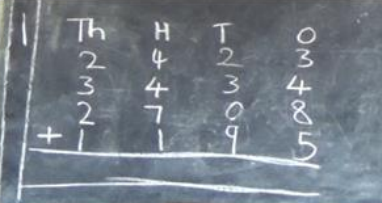
LESSON GRAPH FOR STANDARD 4 LESSON 1	
4¼ min (Whole class)	Asks learners to find $60 + 11$, $70 + 30$, $82 + 12$, $43 + 7$ mentally.
7½ min (Individual work)	Asks the class to workout $3353 + 2122 + 2113 + 1211$ individually in their notebooks. Starts marking. Collects unmarked notebooks.
11½ min (Whole class)	Reviews the meaning of Th, H, T, and O. Reviews the side where addition should start. Solves $3353 + 2122 + 2113 + 1211$ with the class using counters in a plate.
10¾ min (Pair work)	Writes $1432 + 4223$ and $4103 + 3242$ in column layout on the chalkboard under the place-value headings Th H T O. Asks the class to locate the two written problems in their textbooks. Asks them to solve in pairs.
2¼ min (Learners)	Asks 2 learners to write the solutions for $1432 + 4223$ and $4103 + 3242$
17¼ min (Individual work)	Asks learners to solve $2100 + 2232 + 3135$, $1031 + 4315 + 1222$, $2132 + 1326 + 4301 + 1210$, $2414 + 2220 + 3112 + 1133$ from their textbooks. Gives back the notebooks that were collected after the first written task in episode 2. Starts marking. Collects unmarked notebooks.
1¾ min (Class)	Asks learners to solve $56 + 20$ and $14 + 4$ mentally.
LESSON TIME: 55 MINUTES	



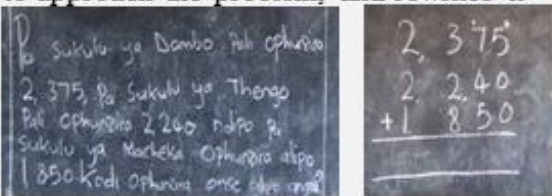
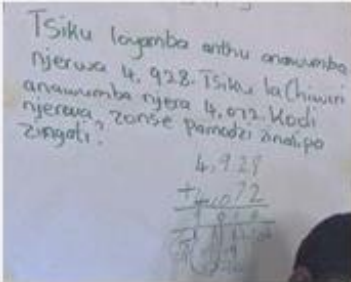
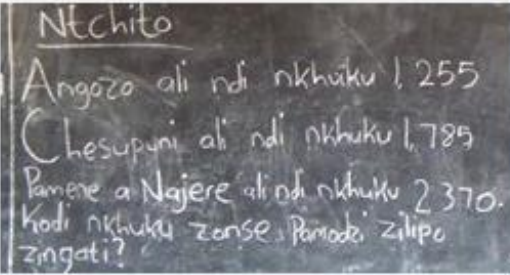
Appendix 18: Lesson graph for Lesson 2 of Standard 4

LESSON GRAPH FOR STANDARD 4 LESSON 2	
6¾ min <i>(Whole class)</i>	Reviews the writing of place-value headings. Asks learners to read :5330, 8679, 7843, 6256. 
16 min <i>(Whole class)</i>	Writes $3165 + 2314 + 3723$ in column layout under the place-value headings Th H T O on the chalkboard. Solves the problem with the class using counters in a plate while recording the regrouping procedure on the chalkboard. 
16½ min <i>(Group work)</i>	Distributes chart papers with pre-written problems such as $1599 + 3383 + 1226$, $1905 + 2589 + 1357$, $1986 + 1749 + 1627$, $2092 + 2014 + 1001$, $2578 + 2057 + 1193$. Learners to put their chart papers on the wall. 
26¾ min <i>(Group work)</i>	Swaps groups to check each if each other's work was done correctly. The group representatives read out the answers to the class. 
4¾ min <i>(Individual work)</i>	Writes $4102 + 1893 + 2016$ and $2431 + 1007 + 3445$ for learners to solve individually in their notebooks.
8¼ min <i>(Learners in front)</i>	Asks two learners to come in front and solve $4102 + 1893 + 2016$ and $2431 + 1007 + 3445$. Gives $1345 + 3316 + 1452 + 1232$ as homework.
LESSON TIME: 1 HOUR 19 MINUTES	

Appendix 19: Lesson graph for Lesson 3 of Standard 4

LESSON GRAPH FOR STANDARD 4 LESSON 3	
8¼ min <i>(Whole class)</i>	Asks learners to find the following sums mentally: $1685 + 1298$ and $6234 + 1398$.
17¾ min <i>(Whole class)</i>	Writes $1450 + 4128 + 2323 + 1979$ in column layout under the place-value headings Th H T O on the chalkboard. Reviews the meaning of Th, H, T, and O with the class. Solves it with the class using counters fitted to a frame. 
3½ min <i>(Group work)</i>	Gives out $5898 + 524$, $5865 + 4075$, $8256 + 1485$, $1549 + 1286$, $1272 + 1764 + 4528$, $3658 + 1278$, $6037 + 1683$, $7289 + 0394$, $3644 + 1789$
31½ min <i>(Group representative s)</i>	Group representatives explain to the class the process they carried out to arrive at the answers to the given problems: $5898 + 524$, $5865 + 4075$, $8256 + 1485$, $1549 + 1286$, $1272 + 1764 + 4528$, $3658 + 1278$, $6037 + 1683$, $7289 + 0394$, $3644 + 1789$. 
11½ min <i>(Individual work)</i>	Writes $2423 + 3434 + 2708 + 1195$ on the chalkboard and asks learners to solve. Goes around to mark. Collects unmarked notebooks. 
6¾ min <i>(A learner in front)</i>	Asks a learner to come in front and solve the problem: $4102 + 1893 + 2016$ and $2431 + 1007 + 3445$. Gives $1345 + 3316 + 1452 + 1232$ for homework
LESSON TIME: 1 HOUR 19 MINUTES	

Appendix 20: Lesson graph for Lesson 4 of Standard 4

LESSON GRAPH FOR STANDARD 4 LESSON 4	
18¾ min (Whole class)	Writes a word problem on the chalkboard involving $2375 + 2240 + 1850$. Discusses with the class how to approach the problem, and rewrites it in place-value layout. Solves it with the class. 
14¾ min (Group work)	Distributes word problems involving $3855 + 1976$, $4928 + 4072$, $1550 + 1350 + 1050 + 1400$, $3442 + 2307 + 2850 + 1328$, $1460 + 2955 + 1178 + 3720$. Learners solve in their groups and paste their solutions on the wall. 
24½ min (Learners representing groups)	Group representatives explain to the class the process they carried out to arrive at the answers to the given problems: $3855 + 1976$, $4928 + 4072$, $1550 + 1350 + 1050 + 1400$, $3442 + 2307 + 2850 + 1328$, $1460 + 2955 + 1178 + 3720$. The teacher and the class compares results from two groups who were given the same problem. 
18¾ min (Individual work)	Writes a word problem leading to $1255 + 1785 + 2370$. Asks learners to solve. Goes around to mark. Collects unmarked notebooks. 
8¾ min (Whole class)	Works out the word problem involving $1255 + 1785 + 2370$ with the class.
LESSON TIME: 1 HOUR 25 MINUTES	

Appendix 21: Interview guide

Interview phase	Items
Setting the atmosphere	• Explain how the data collected will be used. • Ask if the teacher accepts to be interviewed. • Explain why it would be better for the interview to be recorded. • Ask if the teacher accepts that the interview should be recorded.
Teacher's profile	• Overall teaching experience. • Mathematics teaching experience. • Training.
Tasks and examples	• Lesson structure usually followed. • Considerations when deciding the tasks and examples used during the lessons. • Any other considerations that may not have been observed but are often used?
Artefacts	• Selection and preparation of physical teaching resources. • Considerations when deciding the physical artefacts used during the lessons. • Any other artefacts apart from those observed?
Inscriptions	• Reference to specific inscriptions that were observed requiring a further explanation for the choices made. • Reasons behind the choice of medium for inscriptions.
Talk and gesture	• Specific classroom episodes requiring further explanations for the observations made.
Closure	• Reflect on the effect of the presence of the camera in the classroom. • Express gratitude to the teacher for sparing their time for the interview. • Ask if the teacher would welcome any follow-up in the future if need be.

LETTER OF INFORMATION AND CONSENT TO PARTICIPATE IN A RESEARCH STUDY

Introduction

I am a PhD student in Mathematics Education at the University of Malawi, Chancellor College, conducting a study focusing on understanding mediation strategies used by teachers of mathematics in early years classrooms (standards 1 to 4) in Malawi. As part of the study I am expecting to observe a series of mathematics lessons under one topic taught by one teacher in each of the early years classes during the first term of the 2018/2019 academic year.

Participant Consent:

I consent to participate in the study and the following has been explained to me:

- I confirm that I have read and understood the information about this research project. I have had the opportunity to seek clarification and was answered satisfactorily.
- I have been assured that all the information I provide for this study will be treated confidentially and that in any report on the results of this research my identity will remain anonymous.
- I understand that my participation is voluntary and that I can opt out at any time, without necessarily giving a reason for doing so.
- I give my permission for information collected about me to be stored or electronically processed for the purpose of research and to be used in related studies or other studies in the future but only if the research is approved by relevant authorities.
- I understand that participation involves video recordings of my lessons and audio recording of interviews and I have the freedom to access the information I have provided at any time while it is in storage.
- I have been given a copy of the information about this study and this completed consent form for my records.
- I understand that I will not benefit directly from participating in this research.

I voluntarily agree to participate in this study by endorsing my signature below:

 Participant's Name	 Participant's Signature	30/10/18 Date
Fraser Gubede Researcher's Name	 Researcher's Signature	30/10/18 Date

Appendix 23: Request for permission with minuted approval (at the bottom)

Department of Curriculum and Teaching Studies
Chancellor College
P. O. Box 280
Zomba

12th September 2018

The District Education Manager
Zomba Rural

Dear Sir/Madam,

**REQUEST FOR PERMISSION TO CONDUCT A STUDY AT
TWO PRIMARY SCHOOLS IN ZOMBA RURAL**

I am a PhD student in Mathematics Education at Chancellor College, planning to conduct a study at two rural primary schools in Zomba with a record of outstanding performance. The study is focusing on understanding mediation strategies used by teachers of mathematics in early years classrooms (standards 1 to 4). I am expecting to observe a series of lessons taught by one teacher at one school and four teachers at the second school. As a key requirement on research ethics, I will not publish any confidential information and all the study participants will be kept anonymous. As such, I write to request for your permission to conduct the study at these schools during the first and second terms of the 2018/2019 academic year.

I am very hopeful that the findings from this study will inform policy and practice regarding the teaching of mathematics in early years of primary school in Malawi. I will be glad to share the findings from the study with your office if required.

Your assistance will be greatly appreciated.

Yours faithfully



Fraser Gobede

PhD Fellow, NORHED Strengthening Numeracy Project

E-mail: fgobede@cc.ac.mw

Phone: 0888 868 517

please this of ~~Fraser Gobede~~
~~Fraser Gobede~~ accept the
bearer of this letter
to conduct his studies
in your schools.
~~Fraser Gobede~~
for
NORHED

Appendix 24: Letter of introduction



PRINCIPAL
Richard Tambulasi, B.A (Pub Admin), BPA (Hons), MPA, Ph.D

CHANCELLOR COLLEGE
P.O. Box 280, Zomba, Malawi
Telephone: (265) 524 222
Fax: (265) 524 046
E-mail: principal@cc.ac.mw

MATHEMATICS AND SCIENCE EDUCATION SECTION

12th September 2018

The District Education Manager
Zomba Rural

Dear Sir/Madam,

LETTER OF INTRODUCTION: MR FRASER GOBEDE

This is to certify that Mr Fraser Gobede is a PhD (Mathematics Education) student at Chancellor College, University of Malawi. Mr Gobede is required to conduct Mathematics Education research in Malawi primary schools as his doctorate research study. Please assist him accordingly.

If you have any queries or questions regarding Mr Gobede or his research study, then please contact me.

Your assistance will be greatly appreciated

Mercy Kazima Kishindo
Professor of Mathematics Education
Email: mkazima@cc.ac.mw
Tel: 01 524222 Ext 3121

